Warm-up

(1) Find $\log_3 \frac{1}{27} = \log_3 27^{-1} = \log_3 (3^3)^{-1} = \log_3 3^{-3}$

$= \boxed{-3}$

(2) Find $\log_{25} 125$.

By the change-of-base formula: $\log_{25} 125 \overset{⑥}{=} \frac{\ln 125}{\ln 25} = \frac{\ln 5^3}{\ln 5^2}$

$\overset{⑤}{=} \frac{3 \cancel{\ln 5}}{2 \cancel{\ln 5}} = \boxed{\frac{3}{2}}$

(3) $\log_2 50 = \frac{\log 50}{\log 2} = \frac{1.69897}{0.30103} = \boxed{5.64385619}$

[or use ln]

(4) $4^{x+3} = 8^{x-1}$

$(2^2)^{x+3} = (2^3)^{x-1}$

$2^{2x+6} = 2^{3x-3}$

$\log_2 2^{2x+6} = \log_2 2^{3x-3}$

$2x+6 = 3x-3$

$9 = x$

(5) $30 = 100 e^{-.04x}$

$\frac{30}{100} = \frac{100 e^{-.04x}}{100}$

$0.3 = e^{-.04x}$

$\ln 0.3 = \ln e^{-.04x}$

$\ln 0.3 = -.04x$

$\frac{\ln 0.3}{-.04} = \frac{-.04x}{-.04}$

$x = \frac{\ln 0.3}{-.04} = \frac{-1.204}{-.04}$

$= \boxed{30.1}$

Alternate warmup (5) (crazy)

$$30 = 100\, e^{-.04x}$$

$$\ln 30 = \ln\left(100\, e^{-.04x}\right)$$

$$\ln 30 = \ln 100 + \ln e^{-.04x}$$ by the Product Rule

$$\ln 30 - \ln 100 = -.04x$$ by an Inverse property

$$x = \frac{\ln 30 - \ln 100}{-.04} = 30.1$$

why the same? Because $\ln 30 - \ln 100 = \ln\left(\frac{30}{100}\right) = \ln 0.3$

warmup (6)

$$\log_7(x-5) = 2 \rightarrow 7^{\log_7(x-5)} = 7^2$$

$$x - 5 = 7^2$$

$$x - 5 = 49$$

$$x = 54$$

4.5 #44)

$$\log_2(x^2 - x - 2) = 2$$

$$x^2 - x - 2 = 2^2 = 4$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x - 3 = 0 \text{ or } x + 2 = 0$$

$$x = 3 \text{ or } x = -2$$

Are these in the domain of the orginal log function?

That is, is $3^2 - 3 - 2 > 0$? Yes $4 > 0$.

Is $(-2)^2 - (-2) - 2 > 0$? Yes $4 > 0$.

4.3 #50) $\log_3(x+15) - \log_3(x-1) = 2$

<u>Remark</u>: $\log_3(x+15)$ is defined if $x+15>0$
so $x > -15$

$\log_3(x-1)$ is defined if $x-1>0$
so $x>1$

$$\log_3\left(\frac{x+15}{x-1}\right) = 2$$ by the Quotient Law

$$\frac{x+15}{x-1} = 3^2 = 9$$ Multiply by $x-1$:

$$x+15 = 9(x-1)$$
$$x+15 = 9x-9$$
$$24 = 8x$$
$$\boxed{x=3}$$

48) $\log_5 x + \log_5(x+1) = \log_5 20$

<u>Remark</u> we need $x>0$
$x>-1$

$$\log_5[x(x+1)] = \log_5 20$$
$$\log_5(x^2+x) = \log_5 20 \rightarrow 5^{\log_5(x^2+x)} = 5^{\log_5 20}$$
$$x^2+x = 20$$
$$x^2+x-20 = 0$$
$$(x+5)(x-4) = 0$$

Extraneous Solution $\rightarrow$ $\cancel{x=-5}$ or $\boxed{x=4}$

<u>Probable</u> in-class quiz on Monday 3/16 on ch 4

2.5 word problems

Half-Life

$$\boxed{m(t) = m_0\, 2^{-t/h}}$$

$t$ = time

$m(t)$ = mass of material present after $t$ (years, minutes or other unit)

$m_0$ = initial mass

$h$ = half life (units of time)

ex. For polonium-210 $h = 140$ days

$$m(t) = m_0\, 2^{-t/140}$$

| $t$ | $m(t)$ |
|---|---|
| 0 days | $m_0$ |
| 140 | $\frac{1}{2} m_0$ |
| 280 | $\frac{1}{4} m_0$ |
| 420 | $\frac{1}{8} m_0$ |
| 560 | $\frac{1}{16} m_0$ |

scratch work

$m(0) = m_0\, 2^{-0/140} = m_0\, 2^0 = m_0$

$m(140) = m_0\, 2^{-140/140}$

$= m_0\, 2^{-1} = \frac{1}{2} m_0$

$m(280) = m_0\, 2^{-280/140}$

$= m_0\, 2^{-2} = \frac{1}{4} m_0$

Suppose that $m_0 = 100$ g

a) After how many days will $m(t) = 20$ g ?

$$\boxed{m(t) = m_0\, 2^{-t/140}}$$ with $m_0 = 100$ g

$$20 = 100\, 2^{-t/140}$$ and solve for $t$.

$$\frac{20}{100} = \frac{\cancel{100}\, 2^{-t/140}}{\cancel{100}}$$

$$0.2 = \frac{1}{5} = 2^{-t/140}$$

$$\ln 0.2 = \ln 2^{-t/140}$$

$$\ln 0.2 = \left(\frac{-t}{140}\right) \ln 2$$ by the Power Rule

$$\ln 0.2 = \left(\frac{-\ln 2}{140}\right) t$$

$$t = \frac{140 \ln 0.2}{-\ln 2} = 325 \text{ days}$$

b) Find a function $m(t) = m_0 e^{-rt}$ which models the mass.

we want to modify this expression.

~~TRICK~~. $2^{-t/140}$

TRICK: $2 = e^{\ln 2}$

so $2^{-t/140} = \left(e^{\ln 2}\right)^{-t/140} = e^{-\frac{\ln 2}{140} \cdot t}$

So if we take $r = + \frac{\ln 2}{140} = 0.00495$

Then $\boxed{m(t) = 100\, e^{-0.00495 t}}$ describes the same function.

That is, in form $m(t) = m_0 e^{-rt}$

4) Assume population growth is modeled by

$$\boxed{n(t) = n_0 e^{rt}}$$

where
$n_0$ = initial population
$t$ = time
$n(t)$ = population at time $t$
$r$ = relative growth rate

Since the birds were introduced 25 years ago,
At $t = 25$ years, $n(25) = 13{,}000$.
the population doubles every 10 years.

a) what is $n_0$? What was the initial population?

First, find $r$.

$$n(0) = n_0$$
$$n(10) = n_0 e^{r10} = 2n_0$$

$$\frac{2n_0}{n_0} = \frac{n_0 e^{10r}}{n_0}$$

$$2 = e^{10r}$$

$$\ln 2 = \ln e^{10r} = 10r \Rightarrow r = \frac{\ln 2}{10} \approx 0.0693$$

$$\boxed{n(t) = n_0 e^{\left(\frac{\ln 2}{10}\right)t}}$$

Since $n(25) = 13{,}000$

$$13{,}000 = n_0 e^{\left(\frac{\ln 2}{10}\right)\cdot 25}$$

$$13000 = n_0 \cdot 5.6569$$

so $n_0 = \dfrac{13{,}000}{5.6569} = \boxed{2{,}298 \text{ birds}}$ in the initial population.

b) Estimate the population 5 years from now that is, $t = 30$.

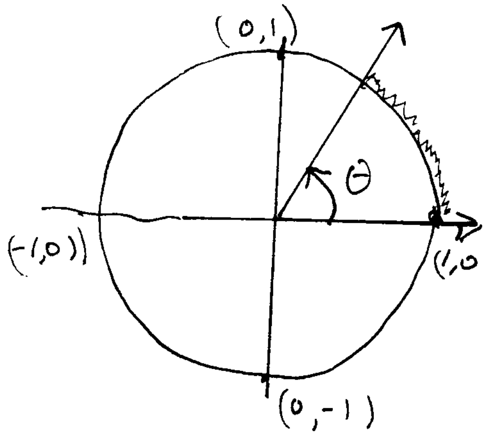
$$n(30) = 2{,}298\, e^{\left(\frac{\ln 2}{10}\right)\cdot 30} = 2{,}298\, e^{3\ln 2}$$

$$= 2{,}298\, e^{\ln 2^3} = 2{,}298\, e^{\ln 8}$$

$$= 2{,}298 \cdot (8) = \boxed{18{,}385 \text{ birds}}$$

## Chap 6.1 – 6.3 and 5.1 – 5.2

### Degree and Radian measure




The length of a string wrapped around a unit circle is the radian measure of the angle.

A unit circle has circumference

$$C = 2\pi \cdot 1 = 2\pi$$

So $360$ degrees $= 2\pi$ radians

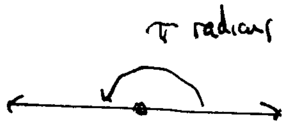
$\boxed{180 \text{ degrees} = \pi \text{ radians}}$

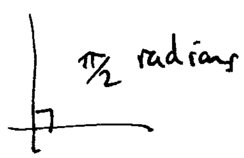
$90$ degrees $= \frac{\pi}{2}$ radians

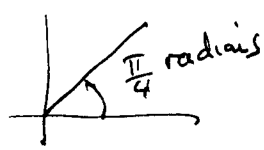
$60$ degrees $= \frac{\pi}{3}$ radians

$45$ degrees $= \pi/4$ radians

$30$ degrees $= \frac{\pi}{6}$ radians



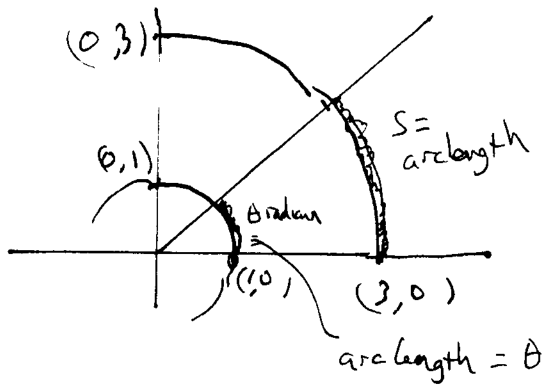




ex: What is 72 degrees in radians?

$$72 \text{ } \cancel{\text{degrees}} \cdot \frac{\pi \text{ radians}}{180 \text{ } \cancel{\text{degrees}}} = \frac{72\pi}{180} = \frac{2\pi}{5} \text{ radians}$$

ex: $1$ radian $= 1 \text{ } \cancel{\text{radian}} \cdot \frac{180 \text{ degrees}}{\pi \text{ } \cancel{\text{radians}}} = \frac{180}{\pi}$ degrees $= 57.3^\circ$



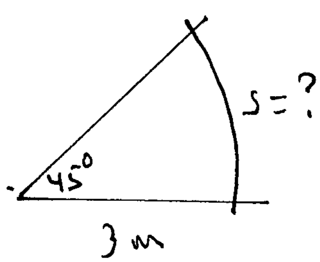

s = arclength $= 3\theta$ where $\theta$ = radian measure of the angle

More generally

$$\boxed{s = r\theta}$$

where $s$ = length of arc (in meters)
$r$ = radius of circle (in meters)
$\theta$ = radian measure of the angle (unitless)

ex: $r = 3$ meters, $\theta = 45°$, what is the arclength?



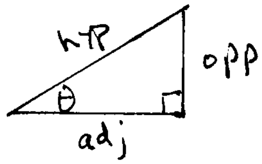

$\theta = \frac{\pi}{4}$ radians

so $s = r\theta = (3 \text{ meters})\left(\frac{\pi}{4}\right) = \frac{3\pi}{4}$ meters

$= 2.36$ meters

Chap 6

Review: Right Triangle Definition of the Trig. Functions




$\sin\theta = \frac{opp}{hyp}$ $\csc\theta = \frac{hyp}{opp}$

$\cos\theta = \frac{adj}{hyp}$ $\sec\theta = \frac{hyp}{adj}$

$\tan\theta = \frac{opp}{adj}$ $\cot\theta = \frac{adj}{opp}$