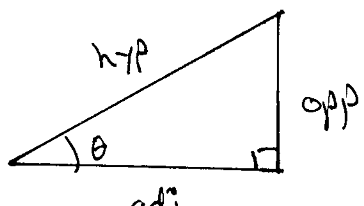
Definitions of trig functions

Right triangle definition (See §6.2) SOH CAH TOA



$$\sin\theta = \frac{opp}{hyp} \qquad \csc\theta = \frac{hyp}{opp}$$

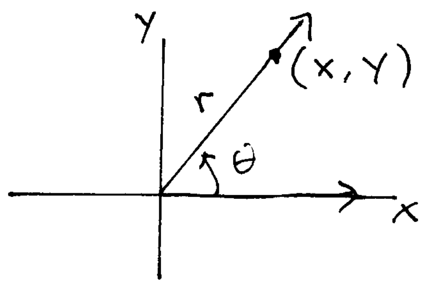
$$\cos\theta = \frac{adj}{hyp} \qquad \sec\theta = \frac{hyp}{adj}$$

$$\tan\theta = \frac{opp}{adj} \qquad \cot\theta = \frac{adj}{opp}$$

Remark: (1) This depends only on the shape of the triangle, not size,

(2) Only works for $0° < \theta < 90°$.

Ratio definition



Take $r = \sqrt{x^2+y^2}$

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

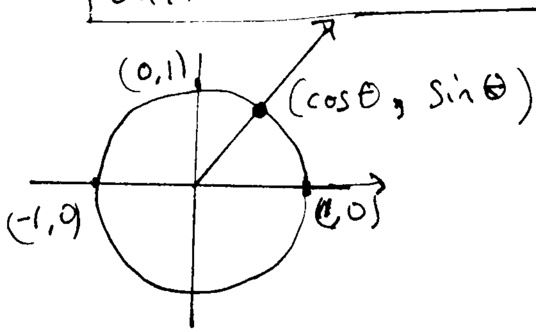
$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Remarks (1) It doesn't matter which point on the terminal ray is (x, y).

(2) With this definition θ can be any real number.

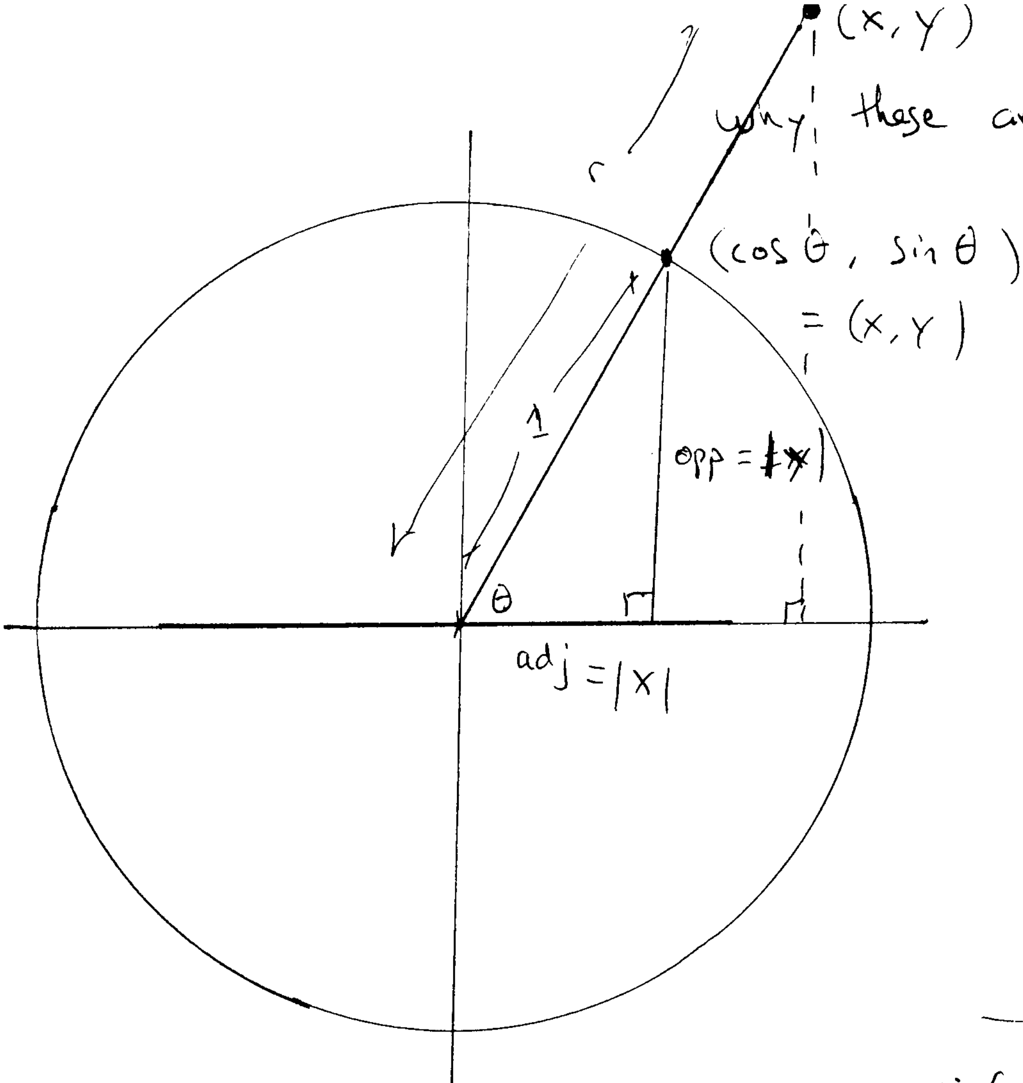
Unit Circle definition



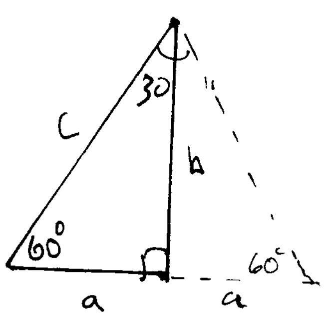
$\cos\theta$ = x-coordinate of the intersection of the terminal ray with the unit circle

$\sin\theta$ = y-coordinate ...

$$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$$



Remark: Remember why:



$a^2 + b^2 = c^2$

And $\boxed{2a = c}$

$a^2 + b^2 = (2a)^2 = 4a^2$

$b^2 = 4a^2 - a^2 = 3a^2$

$\sqrt{b^2} = \sqrt{3a^2} \Rightarrow \boxed{b = a\sqrt{3}}$

Famous angles in Q I

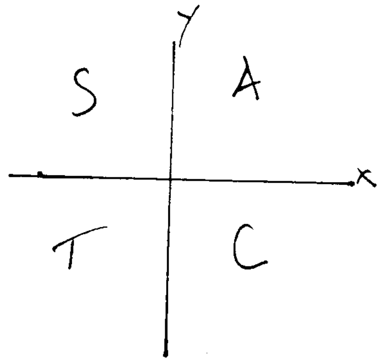
	$\sin\theta$	$\cos\theta$	$\tan\theta$
$0° = 0$	$\frac{\sqrt{0}}{2} = 0$	1	0
$30° = \frac{\pi}{6}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$45° = \frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$60° = \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$90° = \frac{\pi}{2}$	$\frac{\sqrt{4}}{2} = 1$	0	undefined

So $\sin 60° = \frac{b}{c} = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{a}{c} = \frac{a}{2a} = \frac{1}{2}$

M141 3/16/15

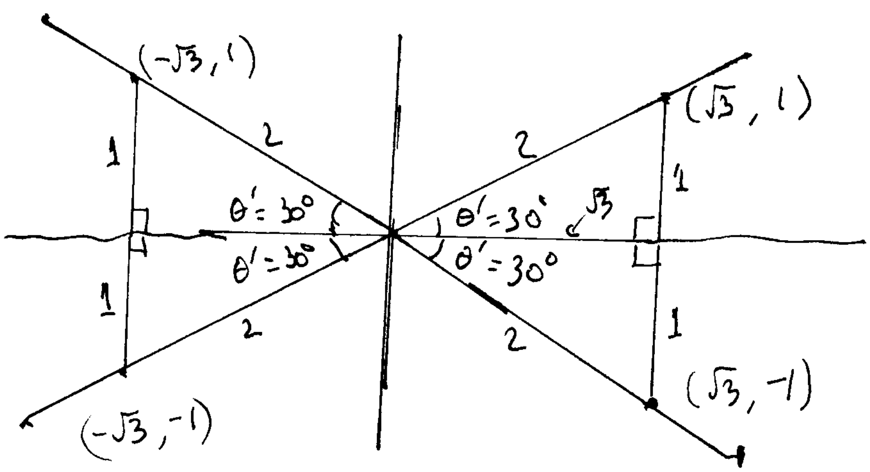
Famous angles in other quadrants?



All
Students
Take
Calculus

All
Strippers
Take
Cash

Reference angles



what are sine cosine and tangent of $\theta = \frac{7\pi}{6}$? $\theta = 210°$?

1. Reference angle? $\theta' = \frac{\pi}{6} = 30°$
2. what quadrant? Q III

$$\sin \frac{7\pi}{6} = -\sin \frac{\pi}{6} = -\frac{1}{2}$$

$$\cos \frac{7\pi}{6} = -\cos \frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

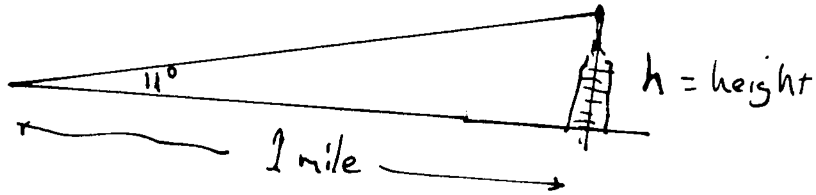
$$\tan \frac{7\pi}{6} = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

m141 3/16/15

(4)

§6.2 #47)

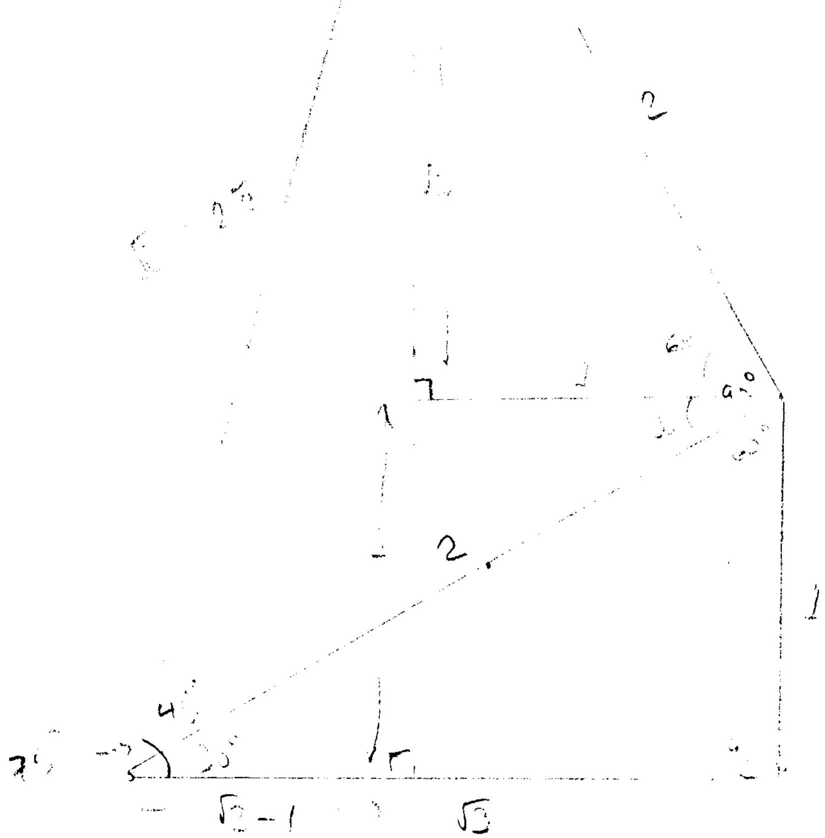
p449



$$\tan 11^\circ = \frac{\text{opp}}{\text{adj}} = \frac{h \text{ feet}}{5280 \text{ ft}}$$

$$h = 5280 \tan 11^\circ = 5280(0.1944)$$
$$= 1026.3 \text{ feet}$$

The Geometry of $15^\circ - 75^\circ - 90^\circ$ triangles



opp of $75^\circ = \sqrt{3}+1$

adj of $75^\circ = \sqrt{3}-1$

hyp = $2\sqrt{2}$

$$\cos 15^\circ = \sin 75^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\sin 15^\circ = \cos 75^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$\rightarrow \frac{4+2\sqrt{3}}{2}$

[illegible] $= \tan 75^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{3+2\sqrt{3}+1}{3-1} = \sqrt{3}+2$

Partial review of inverse trig functions

If θ is an acute angle $0^\circ \le \theta \le 90^\circ$, so All the trig values are positive

then $\theta = \sin^{-1} x$ $= \arcsin x$ means $\sin\theta = x$

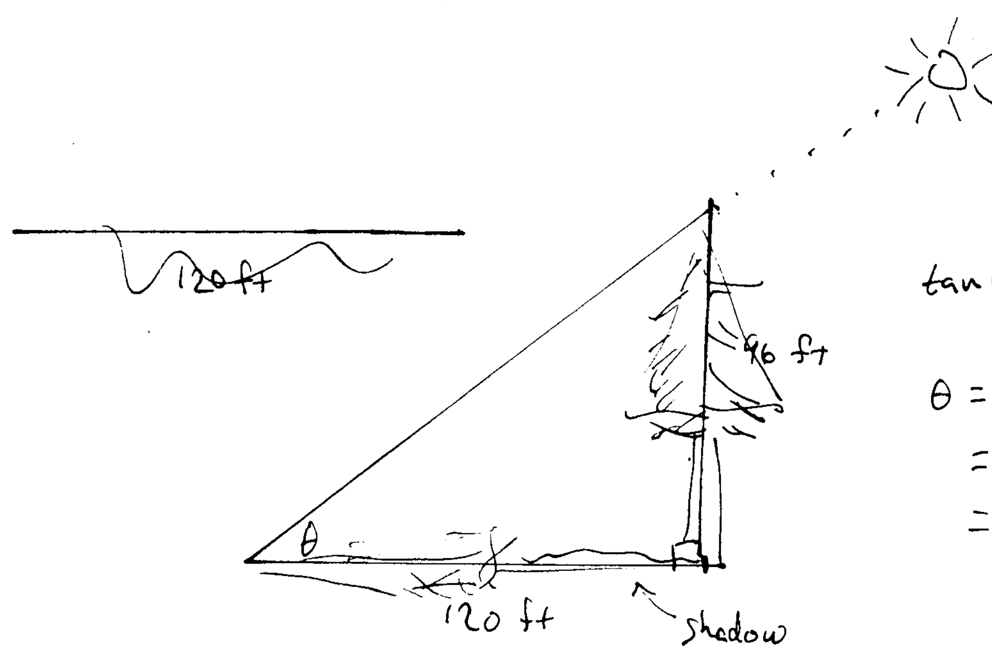
$\theta = \cos^{-1} x$ $= \arccos x$ means $\cos\theta = x$

$\theta = \tan^{-1} x$ $= \arctan x$ mean $\tan\theta = x$

ex: $\arcsin\left(\frac{1}{2}\right) = \sin^{-1}(.5) = 30^\circ = \frac{\pi}{6}$

$\arctan(1) = \tan^{-1}(1) = 45^\circ = \frac{\pi}{4}$

§6.4 #38) p 468 A 96-ft tree casts a shadow that is 120 ft long. what is the angle of elevation of the sun.



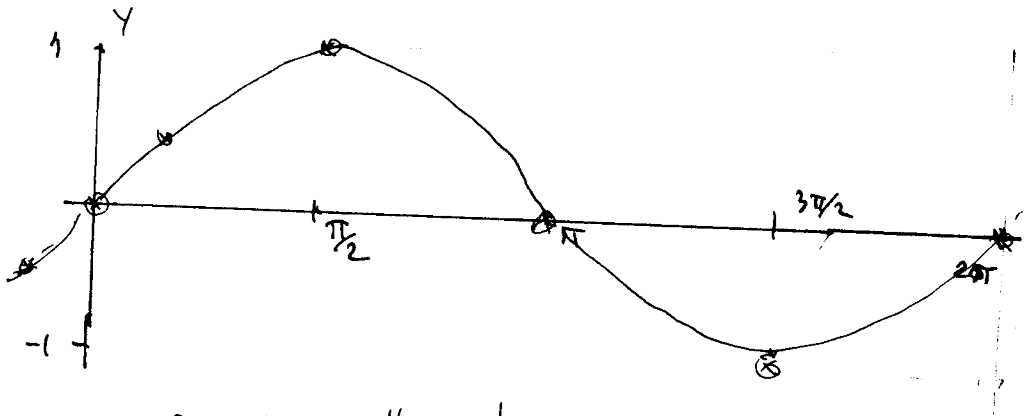
$\tan\theta = \frac{96}{120} = \frac{4}{5} = .8$

$\theta = \arctan(.8)$
$= \tan^{-1}(.8)$
$= 38.66^\circ$

Review of sine and cosine as abstract functions

$y = \sin x$

x	$y = \sin x$
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0



Domain = all reals

Range = $[-1, 1]$

Period = 2π

Amplitude = 1

Sine is an odd function

odd: $f(-x) = -f(x)$