TRIG (continued): Graphs of trig functions [§5.3]

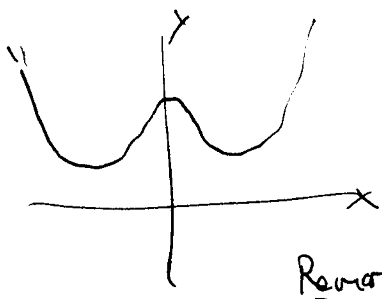
[Neglected topic] Even and Odd Functions

Defn: We say that $g(x)$ is even if $g(-x)=g(x)$

ex: $g(x) = 7x^4+5x^2+3$

$g(-x) = 7(-x)^4+5(-x)^2+3$

$= 7x^4+5x^2+3 = g(x)$



Symmetric across the y-axis.

Remark: If f is any function, then

$$g(x) = \frac{f(x)+f(-x)}{2} \text{ is even}$$

ex: $f(x) = x^2+3x+2$

Let $g(x) = \frac{f(x)+f(-x)}{2} = \frac{(x^2+3x+2)+(x^2-3x+2)}{2}$

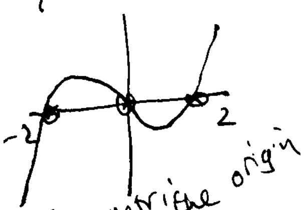
$= \frac{1}{2}\left[2x^2+4\right] = x^2+2$

is an even function.

Defn: We say that $h(x)$ is odd if $h(-x) = -h(x)$.

$= x(x-2)(x+2)$

$y = x^3 - 4x$



example. $h(x) = x^3 - 4x$

$$h(-x) = (-x)^3 - 4(-x) = -x^3 + 4x$$
$$= -(x^3 - 4x) = -h(x)$$

Remark: If f is any function, then

$$h(x) = \frac{f(x) - f(-x)}{2}$$ will be odd.

example. $f(x) = x^2 + 3x + 2$

$$h(x) = \frac{f(x) - f(-x)}{2} = \frac{1}{2}\left[(x^2+3x+2) - (x^2-3x+2)\right]$$
$$= \frac{1}{2}\left[6x\right] = 3x$$

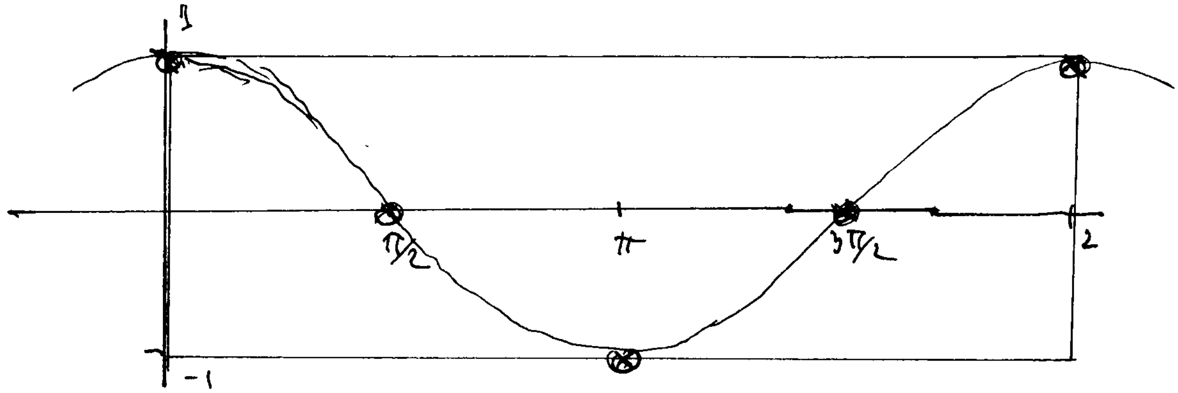
Remark: Odd and even functions are each kind of rare. BUT every function f can be written as the sum of an even part and an odd part. Behold:

$$f(x) = g(x) + h(x) = \underbrace{\left[\frac{f(x)+f(-x)}{2}\right]}_{\text{even}} + \underbrace{\left[\frac{f(x)-f(-x)}{2}\right]}_{\text{odd}}$$

ex: $f(x) = x^4 + 7x^3 + 2x + 5 + \sin x$

$$= \underbrace{\left[x^4 + 5\right]}_{\text{even}} + \underbrace{\left[7x^3 + 2x + \sin x\right]}_{\text{odd}}$$

Graph of $y = \cos x$



Domain = all reals

Range = $[-1, 1]$

Period = 2π

Amplitude = 1

Even function: $\cos(-x) = \cos x$

Summary of general sine and cosine functions

$$\boxed{y = a \sin b(x-c) + d}$$

$|a|$ = amplitude

$\frac{2\pi}{b}$ = period

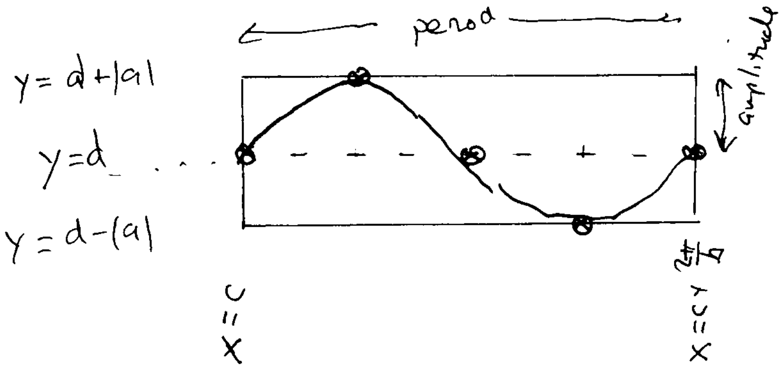
c = phase shift (horizontal shift)

d = vertical shift (or mean value)

Also true for

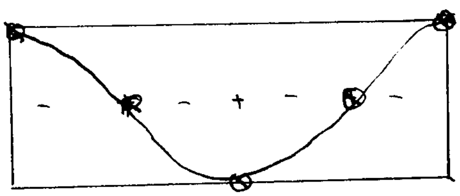
$$\boxed{y = a \cos b(x-c) + d}$$

Case 1 : a is positive



$y = a \sin b(x-c) + d$

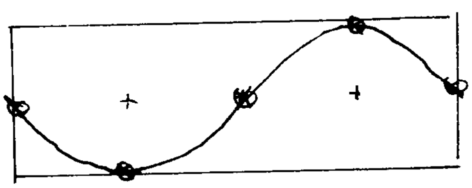
$a>0$



$y = a \cos b(x-c) + d$

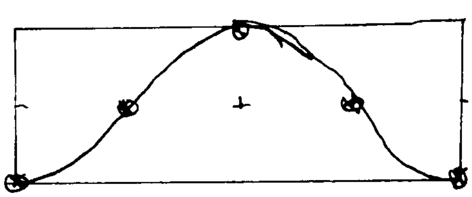
$a>0$

Case 2 : a is negative



$y = a \sin b(x-c) + d$

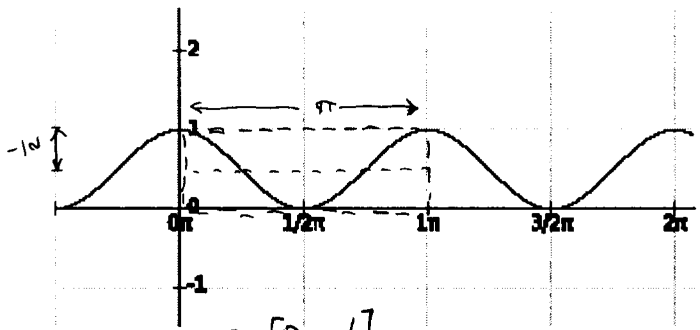
$a<0$



$y = a \cos b(x-c) + d$

$a<0$

exercise: Find another equation for the graph of $y = \cos^2 x$



range $= [0, 1]$

amplitude $= \frac{1}{2} = \frac{1-0}{2}$ $\Rightarrow$ $a = \frac{1}{2}$ or $-\frac{1}{2}$. For the chosen frame, $a = \frac{1}{2}$ and the function is cosine

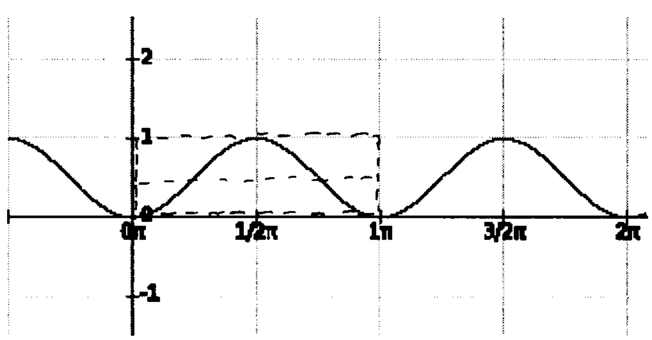
period $= \pi$ $\Rightarrow$ $b = \frac{2\pi}{\pi} = 2$

mean value $= \frac{1}{2} = \frac{1+0}{2}$ $\Rightarrow$ $d = \frac{1}{2}$

phase shift? Depends on frame chosen. Choose $c = 0$

$\therefore$ $\boxed{y = \frac{1}{2}\cos 2x + \frac{1}{2}}$ This suggests the identity $\boxed{\cos^2 x = \frac{1 + \cos 2x}{2}}$

exercise. Find another expression for the graph of $y = \sin^2 x$



range $= [0, 1]$

amplitude $= \frac{1}{2}$ $\Rightarrow$ $a = \frac{1}{2}$ or $-\frac{1}{2}$. For the chose frame, $a = -\frac{1}{2}$ and the function is cosine.

period $= \pi$ $\Rightarrow$ $b = \frac{2\pi}{\pi} = 2$

mean value $= \frac{1}{2}$ $\Rightarrow$ $d = \frac{1}{2}$

phase shift $= 0$ $\Rightarrow$ $c = 0$
(by choice)

$\therefore$ $\boxed{y = -\frac{1}{2}\cos 2x + \frac{1}{2}}$ This suggests the identity $\boxed{\sin^2 x = \frac{1 - \cos 2x}{2}}$