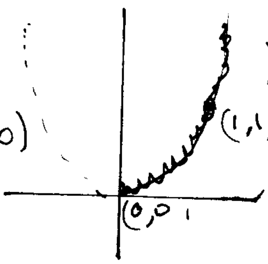
5.4 The other four trig functions - You're on your own

5.5 Inverse trig functions

Let's understand $f(x) = \sqrt{x}$.

Restricted
Domain $= [0, \infty)$
Range $= [0, \infty)$

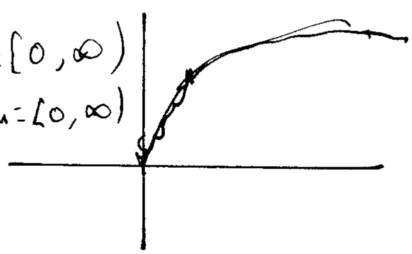


$y = x^2$ is not one-to-one

Restrict the domain to $[0, \infty)$,
So the restricted square is one-to-one.

We define the square-root function to be the inverse.

Range $= [0, \infty)$
Domain $= [0, \infty)$



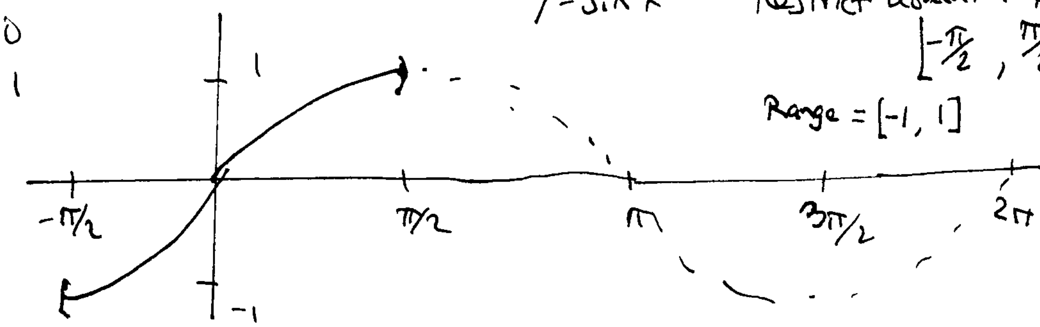
Definition:

$y = \sqrt{x}$ means (1) $y^2 = x$
(2) $y \geq 0$

example: Find $y = \sqrt{49}$
This means (1) $y^2 = 49$ [this says $y = 7$ or $y = -7$]

(2) But $y \geq 0$ so $y = 7$.

x	$\sin x$
$-\pi/2$	-1
0	0
$\pi/2$	1



$y = \sin x$ Restrict domain to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

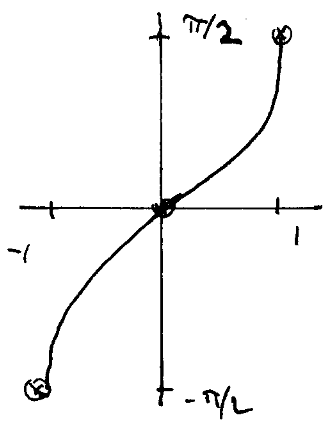
Range $= [-1, 1]$

Call it $y = \text{Sin } x$

Define arcsin to be the inverse.
or $\sin^{-1}$

x	$\arcsin x$
-1	$-\pi/2$
0	0
1	$\pi/2$

$y = \arcsin x = \sin^{-1} x$



Domain $= [-1, 1]$

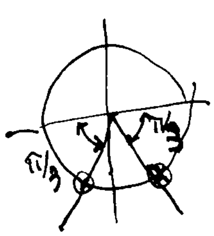
Range $= \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Defintion:

$y = \sin^{-1} x$ means (1) $\sin y = x$

(2) $\underbrace{-\frac{\pi}{2} \le y \le \frac{\pi}{2}}_{\text{QI or QIV}}$

ex: Find $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = y \overset{\text{informally}}{=}$ angle whose sine is $-\frac{\sqrt{3}}{2}$



(1) $\sin y = -\frac{\sqrt{3}}{2}$

Note: Reference angle $y' = 60° = \frac{\pi}{3}$ because $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

So $y = -\frac{\pi}{3}$ or $\cancel{-\frac{2\pi}{3}}$

(2) Also $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ $\qquad y = -\frac{\pi}{3}$