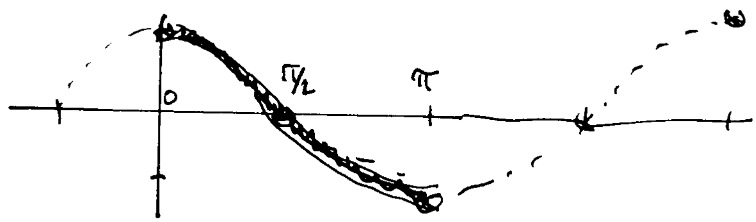
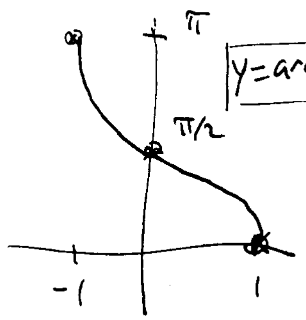
5.5 Inverse Trig functions [cont'd]

$y = \cos x$ domain restricted to $[0, \pi]$



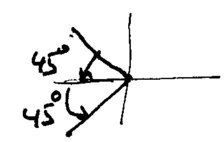
Defn: $y = \arccos x = \cos^{-1} x$ means (1) $\cos y = x$

(2) $0 \le y \le \pi$ ← QI or QII, if $x>0$ / $x<0$



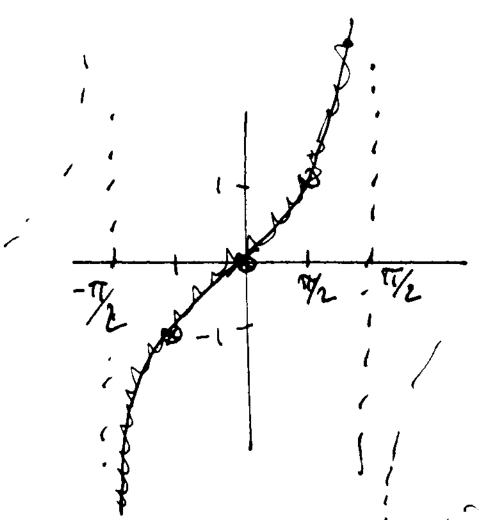
ex: Find $\arccos\left(-\frac{\sqrt{2}}{2}\right) = y$

So (1) $-\frac{\sqrt{2}}{2} = \cos y$



but also (2) $0 \le y \le \pi$

$\therefore\ y = \frac{3\pi}{4}$



$y = \tan x$ Restricted to $(-\frac{\pi}{2}, \frac{\pi}{2})$

Defn: $y = \arctan x = \tan^{-1} x$

means (1) $\tan y = x$

(2) $-\frac{\pi}{2} < y < \frac{\pi}{2}$ ← so y is an "angle" in QI or QIV

Remarks: (1) Arcsine and arctan are odd functions;

$\arcsin(-x) = -\arcsin x$ and $\arctan(-x) = -\arctan x$

Not in textbook

(2) Arccosine is neither even nor odd

but $\arccos(-x) = \pi - \arccos(x)$

(3) $\arccos x = \frac{\pi}{2} - \arcsin x$

Composition of trig and inverse trig functions

Always true: (1) $f(f^{-1}(x)) = x$ $\sqrt[3]{x^3} = x$

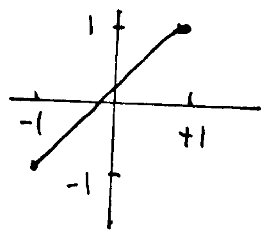
(2) $f^{-1}(f(x)) = x$ e.g. $\log_2 2^x = x$

$10^{\log_{10} x} = x$

ex: $\sin\left(\sin^{-1} \frac{1}{2}\right) = \frac{1}{2}$

$\tan\left(\tan^{-1} \sqrt{3}\right) = \sqrt{3}$

what happens if you graph $y = \sin(\sin^{-1} x)$ on ~~TI-84~~? Graphing calculator?



Answer: $y = x$ but $-1 \le x \le 1$.

ex: $\sin^{-1}\left(\sin \frac{\pi}{6}\right) = \frac{\pi}{6}$

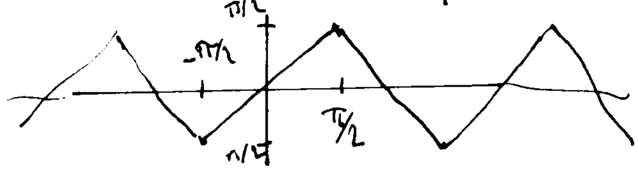
$\tan^{-1}\left(\tan \frac{\pi}{4}\right) = \frac{\pi}{4}$

sneaky $\longrightarrow$ $\sin^{-1}\left(\sin \frac{3\pi}{4}\right) = \frac{\pi}{4}$ whoa! How'd that happen?

Here's why: $\sin \frac{3\pi}{4} = \sin \frac{\pi}{4}$ and what's more, $-\frac{\pi}{2} \le \frac{\pi}{4} \le \frac{\pi}{2}$ ↖ range of arcsine.

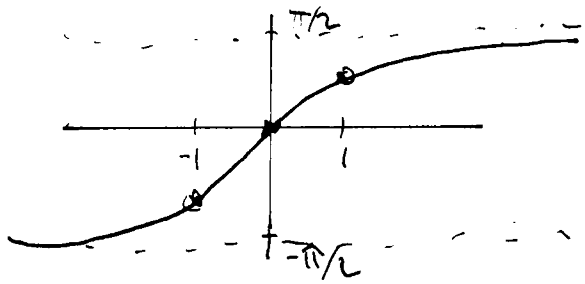
ex Find $\sin^{-1}\left(\sin\left(\frac{7\pi}{4}\right)\right) = \sin^{-1}\left(\sin\left(-\frac{\pi}{4}\right)\right) = -\frac{\pi}{4}$

what happens if we graph $y = \sin^{-1}(\sin(x))$ on a calculator?



Back-up: This is the graph of

$y = \arctan x$



domain = $(-\infty, \infty)$

range = $(-\pi/2, \pi/2)$

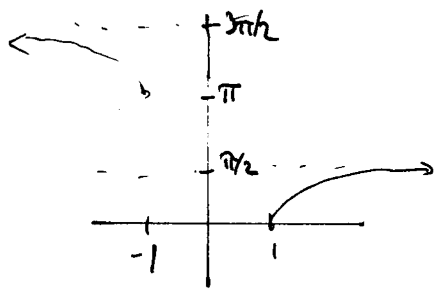
Defn of arcsec

Our textbook: $y = \operatorname{arcsec} x$ means ① $\sec y = x$

and also ② y is in QI (if $x>0$)

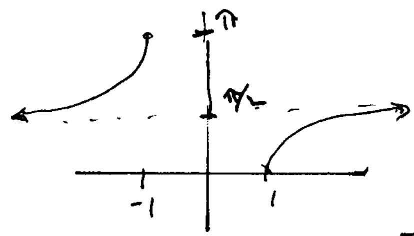
$\rightarrow$ y is in QIII (if $x<0$)

Some other books take the convention y is in QII.



Range = $\{y \mid \cancel{y\leq}\ 0 \leq y < \pi/2$ or $\pi \leq y \leq 3\pi/2\}$

$y = \operatorname{arcsec} x$ for our textbook



Range = $\{y \mid 0 \leq y < \pi/2$ or $\pi/2 < y \leq \pi\}$

$y = \operatorname{arcsec} x$ for other textbooks

Back to composition

§5.5
p412

40) $\cos(\sin^{-1} 0) = \cos\theta$ where $\theta = \sin^{-1} 0$

$\sin\theta = 0$ so $\theta = 0$

$= \cos 0 = 1$

42) $\tan\left(\sin^{-1}\frac{\sqrt{2}}{2}\right) = \tan\theta$ where $\theta = \sin^{-1}\frac{\sqrt{2}}{2}$

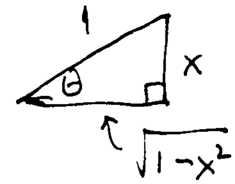
$\sin\theta = \frac{\sqrt{2}}{2}$ so $\theta = \frac{\pi}{4}$

$= \tan\frac{\pi}{4} = 1$

These are from §6.4 on p466

ex: $\cos\left(\sin^{-1}(x)\right) = \cos\theta$ where $\theta = \sin^{-1} x$

$$\sin\theta = \frac{x}{1} = \frac{\text{opp}}{\text{hyp}}$$



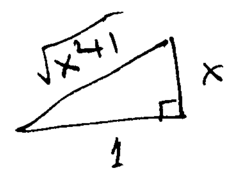
So $\cos\theta = \dfrac{\sqrt{1-x^2}}{1}$

$$= \frac{\text{adj}}{\text{hyp}} = \sqrt{1-x^2}$$

ex: $\sec\left(\tan^{-1}(x)\right)$ where $\theta = \tan^{-1} x$

$$\tan\theta = \frac{x}{1}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{\text{hyp}}{\text{adj}}$$



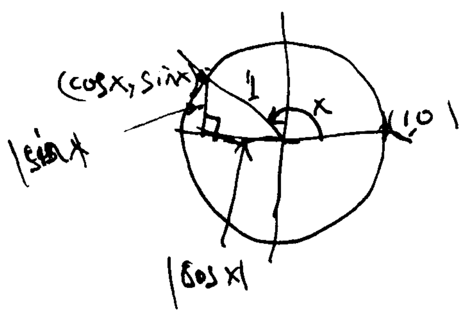
$$= \frac{\sqrt{x^2+1}}{1} = \sqrt{x^2+1}$$

7.1 Fundamental Identities

- Reciprocal identities $\cos x = \dfrac{1}{\sec x}$ etc.
- Quotient " $\tan x = \dfrac{\sin x}{\cos x}$.

- Pythagorean identities

$$\boxed{\sin^2 x + \cos^2 x = 1}$$

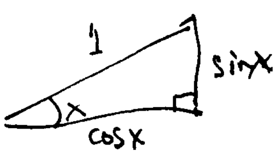


$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\boxed{\tan^2 x + 1 = \sec^2 x}$$

$$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$\boxed{1 + \cot^2 x = \csc^2 x}$$



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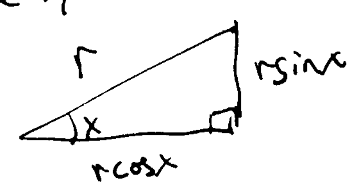
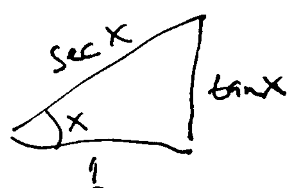
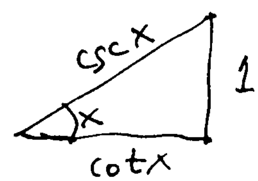






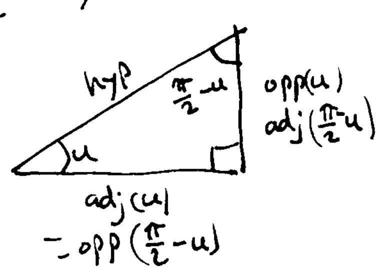
- Even/odd

$\sin(-x) = -\sin x$ $\cos(-x) = \cos x$ $\tan(-x) = -\tan x$

- Cofunction

$\sin\left(\frac{\pi}{2} - u\right) = \cos u$ $\tan\left(\frac{\pi}{2} - u\right) = \cot u$ $\sec\left(\frac{\pi}{2} - u\right) = \csc u$

=



$$\cos u = \frac{\text{adj}(u)}{\text{hyp}} = \frac{\text{opp}(\frac{\pi}{2} - u)}{\text{hyp}} = \sin\left(\frac{\pi}{2} - u\right)$$

Recall: A identity is an equation which is true for all x.

ex: $(x-3)^2 = x^2 - 6x + 9$

Another way to think about identities: a way to say that two functions are the same function

ex. $f(x) = (x-3)^2$ $\qquad g(x) = x^2 - 6x + 9$

Are these the same function? Yes they have the same graph.

ex. Is this an identity? ANSWER: NO

$$\sin 2x = 2\sin x.$$

Let $f(x) = \sin 2x$, and $g(x) = 2\sin x$.

Are they the same? Take $x = \frac{\pi}{6}$.

$$f\left(\frac{\pi}{6}\right) = \sin 2\left(\frac{\pi}{6}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \quad \leftarrow \text{Not equal}$$

$$g\left(\frac{\pi}{6}\right) = 2\sin\left(\frac{\pi}{6}\right) = 2\left(\frac{1}{2}\right) = 1 \quad \leftarrow$$

ex: How about $\cos x = \sqrt{1-\sin^2 x}$? Also, NO.

Let $x = \pi$,

then $\cos\pi = -1$ but $\sqrt{1-\sin^2\pi} = \sqrt{1-0} = 1$

what's going on?

$$\sqrt{1-\sin^2 x} = \sqrt{\cos^2 x} = \sqrt{(\cos x)^2} = |\cos x|$$

So $f(x) = \cos x$

but $g(x) = \sqrt{1-\sin^2 x} = |\cos x|$ are NOT the same function.

<u>Recall</u>: To verify (or prove) ~~and~~ an identity, we must string together <u>known</u> identities (including algebraic identities)

§7.1 p499

76) $$\frac{1+\tan x}{1-\tan x} = \frac{\cos x + \sin x}{\cos x - \sin x}$$

$$LHS = \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}} \cdot \frac{\frac{\cos x}{1}}{\frac{\cos x}{1}} = \frac{\cos x + \frac{\sin x \cdot \cancel{\cos x}}{\cancel{\cos x}}}{\cos x - \frac{\sin x \cdot \cancel{\cos x}}{\cancel{\cos x}}}$$

Quotient ID

algebra

$$= \frac{\cos x + \sin x}{\cos x - \sin x} = RHS$$

68) $$\frac{1-\cos x}{\sin x} + \frac{\sin x}{1-\cos x} = 2\csc x$$

$(1-\cos x)^2$

$$LHS = \frac{(1-\cos x)(1-\cos x)}{\sin x\,(1-\cos x)} + \frac{\sin x \cdot \sin x}{(1-\cos x)\cdot \sin x}$$

$$= \frac{1-2\cos x+\cos^2 x + \sin^2 x}{\sin x(1-\cos x)}$$

using algebra: $(a-b)^2 = a^2-2ab+b^2$ with $a=1$, $b=\cos x$.

Pythagorean ID

$$= \frac{1-2\cos x+1}{\sin x(1-\cos x)} = \frac{2-2\cos x}{\sin x(1-\cos x)}$$

$$= \frac{2\cancel{(1-\cos x)}}{\sin x\cancel{(1-\cos x)}} = \frac{2}{\sin x} = 2\cdot\frac{1}{\sin x} = 2\csc x$$

Reciprocal ID