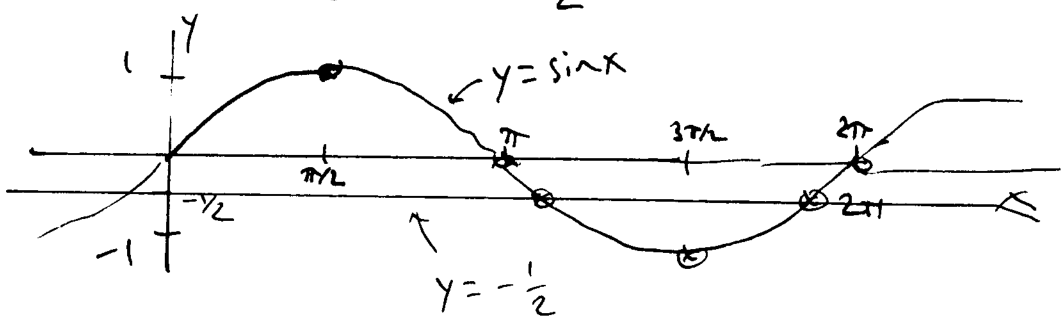
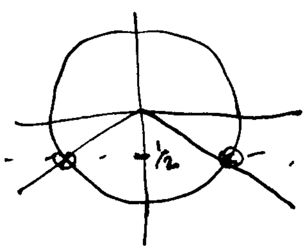
7.4 Basic Trig Equations

ex: $2\sin x + 1 = 0$

$$2\sin x = -1$$

$$\sin x = -\frac{1}{2}$$





Step 1 solve $\sin\theta' = \left|-\frac{1}{2}\right| = \frac{1}{2}$

$$\theta' = \arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$$ ← Between 0° and 90°

By calculator, if necessary

Step 2 Now worry about the quadrant.

Since sine is negative in QIII and QIV find angles in those two quadrants with reference angle $\frac{\pi}{6}$, Namely,

$$\pi + \frac{\pi}{6} = \frac{7\pi}{6} \leftarrow \text{in QIII}$$

or $$2\pi - \frac{\pi}{6} = \frac{11\pi}{6} \leftarrow \text{in QIV}.$$

Step 3 If we want all possible solutions, use that sine has period 2π:

$$x = \frac{7\pi}{6} + 2k\pi$$

OR

$$x = \frac{11\pi}{6} + 2k\pi$$

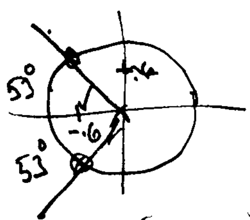
ex: Find all solutions in degrees of $\cos\theta = -0.6$

Step 1 Solve, in $\cancel{[0, \pi/2]}$ $[0, 90^\circ]$, $\cos\theta' = 0.6$

$$\theta' = \arccos(0.6) = 53.13^\circ$$

Step 2 where is cosine negative? QII and QIII

Use this to find solutions in $[0^\circ, 360^\circ]$



$$\theta = 180^\circ - 53.13^\circ = 126.87^\circ \leftarrow \text{QII}$$

or $\theta = 180^\circ + 53.13^\circ = 233.13^\circ \leftarrow \text{QIII}$

Step 3 General solutions

$$\theta = 126.87^\circ + 360k^\circ$$

or $\theta = 233.13^\circ + 360k^\circ$

where k is any integer: $k = \ldots -2, -1, 0, 1, 2, \ldots$

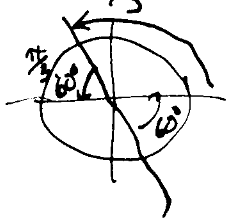
ex: Find all solutions of $\tan x = -\sqrt{3}$.

Step 1 Solve $\tan x' = \sqrt{3}$, in $[0, \frac{\pi}{2})$

$$x' \, [= 60^\circ] = \frac{\pi}{3}$$

Step 2 where is tangent negative? QII and QIV.

$$x = \frac{2\pi}{3} \quad \left[\text{or } x = \frac{5\pi}{3}\right]$$



Step 3 General solution $x = \frac{2\pi}{3} + k\pi$

because tangent has period π.

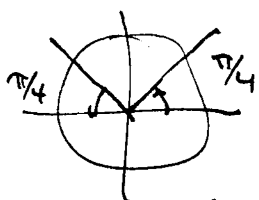
Quadratic Trig Equations

ex: $\sin^2 x = \frac{1}{2}$

$$\sin x = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$$

$$\sin x = \frac{\sqrt{2}}{2} \quad \text{OR} \quad \sin x = -\frac{\sqrt{2}}{2}$$

case 1

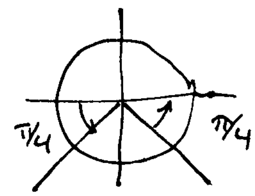


$x = \frac{\pi}{4} + 2k\pi$

or

$x = \frac{3\pi}{4} + 2k\pi$

case 2



$x = \frac{5\pi}{4} + 2k\pi$

or

$x = \frac{7\pi}{4} + 2k\pi$

7.4 47) $\cos^2\theta - \cos\theta - 6 = 0$ Find all solutions (in radians)

Let $u = \cos\theta$.

$$u^2 - u - 6 = 0$$

$$(u+2)(u-3) = 0$$

$$u+2=0 \quad \text{or} \quad u-3=0$$

$$u=-2 \quad \text{or} \quad u=3$$

$$\cos\theta = -2 \quad \text{or} \quad \cos\theta = 3$$

No solution because the range of $\cos\theta$ is $[-1, 1]$.

7.4 44) $\tan^4\theta - 13\tan^2\theta + 36 = 0$ Let $u = \tan\theta$. [in Radians]

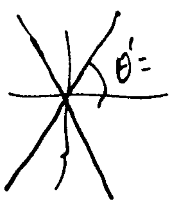
$$u^4 - 13u^2 + 36 = 0$$

$$(u^2-4)(u^2-9) = 0$$

$$(u-2)(u+2)(u-3)(u+3) = 0$$

$$u = 2 \text{ or } u = -2 \text{ or } u = 3 \text{ or } u = -3$$

$$\tan\theta = 2 \text{ or } \tan\theta = -2 \text{ or } \tan\theta = 3 \text{ or } \tan = -3$$



$\theta' =$

$\theta' = \tan^{-1}(2)$

$= 1.1 \ (= 63.4^\circ)$

$\theta = \boxed{1.1 + k\pi}$

$(= 63.4^\circ + 180^\circ k)$

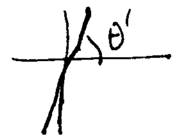
$\theta' = 1.1 \ (= 63.4^\circ)$

$\theta' = (\pi - 1.1) + k\pi$

$(= 180^\circ - 63.4^\circ) + 180^\circ k$

$= \boxed{2.0 + k\pi}$

$(= 116.6^\circ + 180^\circ k)$

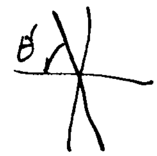


θ'

$\theta' = \tan^{-1}(3)$

$= 1.25$

$\theta = \boxed{1.25 + k\pi}$



θ

$\theta' = \tan^{-1}(3)$

$= 1.25$

$\theta = \pi - 1.25 = 1.89$

$\theta = \boxed{1.89 + k\pi}$

7.5 More trig equations - Using identities to help solve trig eqns

10) $\cos 2\theta = \cos^2\theta - \frac{1}{2}$ Use a double-angle identity

$2\cos^2\theta - 1 = \cos^2\theta - \frac{1}{2}$ Let $u = \cos\theta$.

$$2u^2 - 1 = u^2 - \frac{1}{2}$$

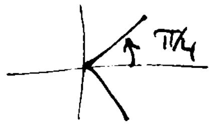
$$u^2 - 1 = -\frac{1}{2}$$

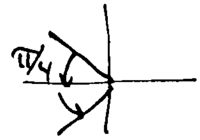
$$u^2 = \frac{1}{2}$$

$$\cos^2\theta = \frac{1}{2}$$

$$\cos\theta = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$$

(10 cont'd) $\cos\theta = \frac{\sqrt{2}}{2}$ or $\cos\theta = -\frac{\sqrt{2}}{2}$





$\theta = \frac{\pi}{4} + 2k\pi$

$\frac{3\pi}{4} + 2k\pi$

$\frac{3\pi}{4} + 2k\pi$

$\frac{5\pi}{4} + 2k\pi$

7.5 11) $2\sin^2\theta - \cos\theta = 1$ Use $\sin^2\theta = 1 - \cos^2\theta$

$2(1-\cos^2\theta) - \cos\theta = 1$

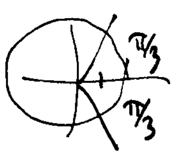
$2 - 2\cos^2\theta - \cos\theta = 1$

$0 = 2\cos^2\theta + \cos\theta - 1$ Let $u = \cos\theta$ or factor directly

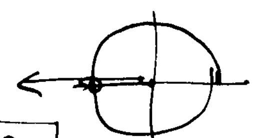
$0 = (2\cos\theta - 1)(\cos\theta + 1)$

$2\cos\theta - 1 = 0$ or $\cos\theta + 1 = 0$

$\cos\theta = \frac{1}{2}$ or $\cos\theta = -1$



$$\boxed{\begin{array}{c}\theta = \frac{\pi}{3} + 2k\pi \\ \text{or} \\ \theta = -\frac{\pi}{3} + 2k\pi\end{array}}$$



$$\boxed{\theta = -\pi + 2k\pi}$$

Fancy compact expression of the solution:

$$\boxed{\theta = \frac{\pi}{3} + \frac{2\pi}{3}n}$$ $n = \ldots -2, -1, 0, 1, 2, \ldots$

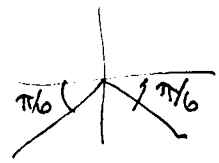
7.5 20) ~~28)~~ [Involves $\sin(3\theta)$ not $\sin\theta$]

a) Find all solutions
b) Solutions in $[0, 2\pi)$

$$2\sin 3\theta + 1 = 0$$

$$\sin 3\theta = -\frac{1}{2}$$

Let u = angle $= 3\theta$



$$\sin u = -\frac{1}{2}$$

method:
First solve for the angle

$$3\theta = u = \frac{7\pi}{6} + 2k\pi$$

OR

$$3\theta = u = \frac{11\pi}{6} + 2k\pi$$

Secondly, solve for the variable

$$\theta = \frac{1}{3}\left[\frac{7\pi}{6} + 2k\pi\right] = \boxed{\frac{7\pi}{18} + \frac{2k\pi}{3}}$$

OR

$$\theta = \frac{1}{3}\left[\frac{11\pi}{6} + 2k\pi\right] = \boxed{\frac{11\pi}{18} + \frac{2k\pi}{3}}$$

Answer to a)

b) Solutions for $0 \le \theta < 2\pi$ so $0 \le 3\theta < 6\pi$

That is, expect Six solutions

$$\theta = \boxed{\frac{7\pi}{18}}, \boxed{\frac{11\pi}{18}}, \frac{7\pi}{18} + \frac{12\pi}{18} = \boxed{\frac{19\pi}{18}}, \frac{11\pi}{18} + \frac{12\pi}{18} = \boxed{\frac{23\pi}{18}}$$

$$\frac{19\pi}{18} + \frac{12\pi}{18} = \boxed{\frac{31\pi}{18}}, \frac{23\pi}{18} + \frac{12\pi}{18} = \boxed{\frac{35\pi}{18}}$$

Solutions $[0, 2\pi)$:

46) $\tan\theta + \cot\theta = 4\sin 2\theta$

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 4 \cdot 2\sin\theta\cos\theta = 8\sin\theta\cos\theta$$

multiply both sides by $\sin\theta\cos\theta$:

$$\frac{\sin\theta \cdot \sin\theta \cancel{\cos\theta}}{\cancel{\cos\theta}} + \frac{\cos\theta\ \cancel{\sin\theta}\cos\theta}{\cancel{\sin\theta}} = 8\sin^2\theta\cos^2\theta$$

46 cont'd)

$$\sin^2\theta + \cos^2\theta = 8\sin^2\theta\cos^2\theta$$

$$1 = 8\sin^2\theta\cos^2\theta$$

$$1 = 8\sin^2\theta(1-\sin^2\theta)$$

$$1 = 8\sin^2\theta - 8\sin^4\theta$$

$$8\sin^4\theta - 8\sin^2\theta + 1 = 0 \qquad \text{Let } u = \sin^2\theta.$$

$$8u^2 - 8u + 1 = 0$$

$$u = \frac{8 \pm \sqrt{64-32}}{16} = \frac{8 \pm \sqrt{32}}{16} = \frac{8 \pm 4\sqrt{2}}{16}$$

$$= \frac{8}{16} \pm \frac{4\sqrt{2}}{16} = \frac{1}{2} \pm \frac{\sqrt{2}}{4}$$

BETTER METHOD

$$\sin^2\theta = u = \frac{1}{2} + \frac{\sqrt{2}}{4} \quad \text{or} \quad \sin^2\theta = u = \frac{1}{2} - \frac{\sqrt{2}}{4}$$

$$= \frac{2+\sqrt{2}}{4} \qquad\qquad = \frac{2-\sqrt{2}}{4}$$

$$\sin\theta = \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2} \text{ or } \sin\theta = -\frac{\sqrt{2+\sqrt{2}}}{2} \text{ or } \sin\theta = \frac{\sqrt{2-\sqrt{2}}}{2} \text{ or } \sin\theta = -\frac{\sqrt{2-\sqrt{2}}}{2}$$

$\sin\theta = 0.92388$ etc.

A better way to solve 7.5 46) (written after class)

To be finished later

$$\tan\theta + \cot\theta = 4\sin 2\theta$$

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 4\sin 2\theta \qquad \text{Multiply both sides by } \sin\theta\cos\theta\text{:}$$

pythagorean ID

$$\sin^2\theta + \cos^2\theta = 4\sin 2\theta\sin\theta\cos\theta = 2\cdot\sin 2\theta\cdot(2\sin\theta\cos\theta)$$ Double angle ID

$$1 = 2\sin 2\theta\cdot\sin 2\theta = 2\sin^2 2\theta$$ power reducing ID

$$1 = 2\left(\frac{1-\cos 2(2\theta)}{2}\right) = 1 - \cos 4\theta$$

$$0 = \cos 4\theta$$

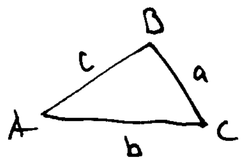
$$4\theta = \frac{\pi}{2} + 2k\pi \quad \text{or} \quad 4\theta = \frac{3\pi}{2} + 2k\pi$$

$$\theta = \frac{1}{4}\left[\frac{\pi}{2} + 2k\pi\right] = \frac{\pi}{8} + \frac{4\pi k}{8} \quad \text{or} \quad \theta = \frac{1}{4}\left[\frac{3\pi}{2} + 2k\pi\right] = \frac{3\pi}{8} + \frac{4k\pi}{8}$$

In $[0, 2\pi)$: $\boxed{\theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}}$

Intro to

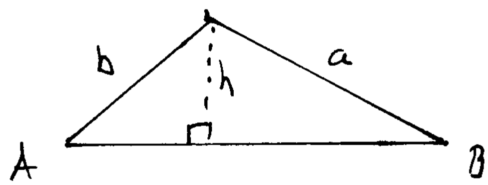
6.5 and 6.6 Law of Sines and Law of Cosines



$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$ Law of Sines

$$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$$ Law of Cosines

<u>proof</u> of Law of Sines



$\frac{\text{opp}}{\text{hyp}} = \frac{h}{b} = \sin A$ also $\frac{h}{a} = \sin B$

$h = b\sin A$ $h = a\sin B$

$b\sin A = a\sin B$ Divide by ab

$$\frac{\cancel{b}\sin A}{a\cancel{b}} = \frac{\cancel{a}\sin B}{\cancel{a}b}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

<u>Semi-official</u> declaration of what will be on <u>Test 3</u> of Monday April 20 (and homework due):

5.4 more trig graphs

5.5, 6.4 } Inverse trig functions, solving right triangles

7.1–7.3 Trig identities

7.4–7.5 Conditional trig equations

6.5, 6.6 } Law of sines, " " cosines, solving triangles

For this test <u>NO NOTES</u>.

Memorize (or be prepared to "buy") all identities up to and including HALF ANGLE identities (so <u>NOT</u> product-to-sum or sum-to-product)