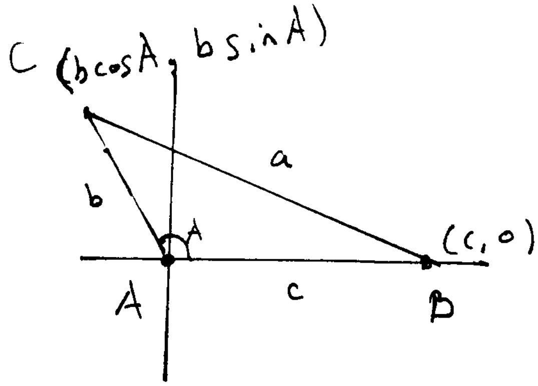
6.5 and 6.6 How to ~~the~~ Derive the Law of Cosines



Ingredients:
- Distance formula (Pythagorean thm)
- Pythagorean identity
- Algebra

<u>Note:</u> $\cos A = \dfrac{x}{r} = \dfrac{x}{b}$ $\quad$ $\sin A = \dfrac{y}{r} = \dfrac{y}{b}$

so $x = b\cos A$ $\quad$ $y = b\sin A$

Now find $a^2 = \text{distance}^2$ between vertex B and vertex C.

$$a^2 = (b\cos A - c)^2 + (b\sin A - 0)^2$$

← square of the Distance Formula

$$= b^2\cos^2 A - 2bc\cos A + c^2 + b^2\sin^2 A$$
$$= b^2\underbrace{(\sin^2 A + \cos^2 A)}_{1} + c^2 - 2bc\cos A$$

$$a^2 = b^2 + c^2 - 2bc\cos A.$$

<u>Remark:</u> we can permute a, b and c to get the other forms

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

<u>Remark:</u> If $C = 90^\circ$, then $\cos C = \cos 90^\circ = 0$, we get

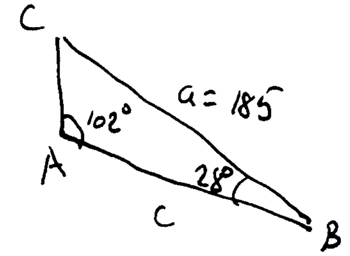
$c^2 = a^2 + b^2$ so the Pythagorean Theorem is a special case of the Law of Cosines.

<u>ASA or AAS case</u> – Use Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

6.5 8)

$\boxed{AAS}$



$a = 185$

$B = 28°$

$A = 102°$

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$b = \frac{a \sin B}{\sin A} = \frac{185 \sin 28°}{\sin 102°} = \boxed{88.8}$$

$$C = 180° - 102° - 28° = \boxed{50°}$$

$$\frac{c}{\sin C} = \frac{a}{\sin A} \Rightarrow c = \frac{a \sin C}{\sin A} = \frac{185 \sin 50°}{\sin 102°} = \boxed{144.9}$$

<u>ex</u> $\boxed{SSA}$ Theme – Let the algebra be our guide.

20) $a = 30$

$c = 40$

SSA $\angle A = 37°$

$$\frac{\sin A}{a} = \frac{\sin C}{c} \Rightarrow \sin C = \frac{c \sin A}{a}$$

$$\Rightarrow \sin C = \frac{40 \sin 37°}{30} = 0.80242$$

<u>Two Solutions</u>: $C_1 = \sin^{-1} 0.80242 = \boxed{53.36°}$ (in QI)

<u>also</u> $C_2 = 180° - 53.36° = \boxed{126.64°}$ (in QII)

If $C_1 = 53.36°$ then $B_1 = 180° - 37° - 53.36° = \boxed{89.64°}$

If $C_2 = 126.64°$ then $B_2 = 180° - 126.64° - 37° = \cancel{89.64°}\ \boxed{16.36°}$

$$\frac{b}{\sin B} = \frac{a}{\sin A} \Rightarrow b = \frac{a \sin B}{\sin A}$$

so $\begin{cases} \text{If } B_1 = 89.64° \text{ then } b_1 = \dfrac{30 \sin 89.64°}{\sin 37°} = \boxed{49.8} \\ \text{If } B_2 = 16.36° \text{ then } b_2 = \dfrac{30 \sin 16.36°}{\sin 37°} = \boxed{14.0} \end{cases}$

MH1 4/15/15

3

[Do as much as possible without thinking.]

6.5 25)

$\boxed{SSA}$

$a = 50$

$b = 100$

$\angle A = 50^\circ$

$$\frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow \sin B = \frac{b \sin A}{a}$$

$$= \frac{100 \sin 50^\circ}{50}$$

$$= 1.53 \quad \text{and STOP}$$

$\boxed{\text{No solution}}$.

ex $\boxed{SAS}$

6.6 4) [Let's see how much we can do without thinking.]

$b = 15$ ft

$c = 18$ ft

$\angle A = 108^\circ$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$= 15^2 + 18^2 - 2(15)(18) \cos 108^\circ$$

$$= 715.87 \Rightarrow a = \sqrt{715.87}$$

$$a = \boxed{26.76} \text{ ft}$$

Strategy: We would like to avoid a sine equation with two solutions. Idea: A triangle has, at most, one obtuse angle ($> 90^\circ$). The strategy at this point is to look for the smaller of two angles, for it will be acute. Look for B:

$$\frac{\sin B}{b} = \frac{\sin A}{a} \Rightarrow \sin B = \frac{b \sin A}{a} = \frac{15 \sin 108^\circ}{26.76} = 0.533$$

While this sine equation has two solutions we only need the QI solution, so $B = \sin^{-1}(0.533) = \boxed{32.2^\circ}$

$$\angle C = 180^\circ - 108^\circ - 32.2^\circ = \boxed{39.8^\circ}$$

ex: $\boxed{SSS}$

10) $a = 12$ m
$b = 10$ m
$c = 20$ m

Strategy: Find the largest angle angle, so if it's obtuse the remaining two will be acute. So Find $\angle C$.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$2ab\cos C = a^2 + b^2 - c^2$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{12^2 + 10^2 - 20^2}{2(12)(10)} = -0.65$$

Aha! C is obtuse. So $\angle C = \cos^{-1}(-0.65) = \boxed{130.5^\circ}$

Now use Law of Sines without worry of two solutions in the sine equation ... [To be finished in notes]

written [After class]

$$\frac{\sin A}{a} = \frac{\sin C}{c} \Rightarrow \sin A = \frac{a \sin C}{c} = \frac{12 \sin 130.5^\circ}{20} = 0.456$$

Normally we would expect this sine equation to have a QI solution and a QII solution, but since we know A must be acute (for $\angle C > 90^\circ$), we only need the quadrant I solution: $A = \sin^{-1}(0.456) = \boxed{27.1^\circ}$

Finally, $\angle B = 180^\circ - \angle A - \angle C = 180^\circ - 27.1^\circ - 130.5^\circ = \boxed{22.3^\circ}$

More NOTES on what may be on Test 3 on Monday, Apr 20

The HOMEWORK which is due: 5.4 – 5.5
6.4 – 6.6
7.1 – 7.5

The TEST will (or may) cover these sections:

5.4 Graphs of tan, sec, csc [will not be on the test]

5.5, 6.4 } Definition and properties of $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ functions
ex: Find $\tan^{-1}(-\sqrt{3})$ without using a calculator
ex: Rewrite as an algebraic expression in x: $\cos(\sin^{-1} x)$ [see §6.4 #33]

7.1 Basic trig identities; proving identities
7.2 Addition, subtraction identities
7.3 Double angle, Power-reducing, Half-angle [.. product-to-sum] Not on test
7.4 Trig equations – including equations which are quadratic in form
7.5 Trig equations – which may require identities to rewrite first

6.5 Law of sines: ASA, AAS case; SSA case (possibly ambiguous).
6.6 Law of cosines: SAS case; SSS case.

IDENTITIES you must know, and the section where they first appear.

§ 7.1
- Reciprocal
- Quotient
- Pythagorean
- Even / Odd
- Cofunction

§ 7.2
- Addition, subtraction (of sin, cos, tan)

§ 7.3
- Double angle
- Power-reducing
- Half angle

[You may ignore: • Product-to-sum • Sum-to-product]

§ 6.5 • Law of Sines
§ 6.6 • Law of Cosines