8.3 Polar form of complex numbers (finishing)

ex: what are the four 4th roots of -4?

By the previous calculations $1+i$ is a 4th root.

$$-4 = 4(\cos 180° + i \sin 180°)$$
$$= 4(\cos(180°+360°) + i \sin(180°+360°))$$
$$= 4[\cos(180°+720°) + i \sin(180°+720°)]$$
$$= 4[\cos(180°+1080°) + i \sin(180°+1080°)]$$
$$\vdots$$

Now take the 4th roots of each of the polar forms,

$$w_1 = 4^{1/4}\left[\cos\left(\frac{180°}{4}\right) + i \sin\left(\frac{180°}{4}\right)\right] = \boxed{\sqrt{2}(\cos 45° + i \sin 45°)}$$

$$w_2 = 4^{1/4}\left[\cos\left(\frac{180°}{4} + \frac{360°}{4}\right) + i \sin\left(\frac{180°}{4} + \frac{360°}{4}\right)\right]$$
$$= \boxed{\sqrt{2}[\cos(45°+90°) + i \sin(45°+90°)]}$$

$$w_3 = \boxed{\sqrt{2}[\cos(45°+180°) + i \sin(45°+180°)]}$$

because $\frac{720°}{4} = \frac{2 \cdot 360°}{4} = 2 \cdot 90° = 180°$

$$w_4 = \boxed{\sqrt{2}[\cos(45°+270°) + i \sin(45°+270°)]}$$

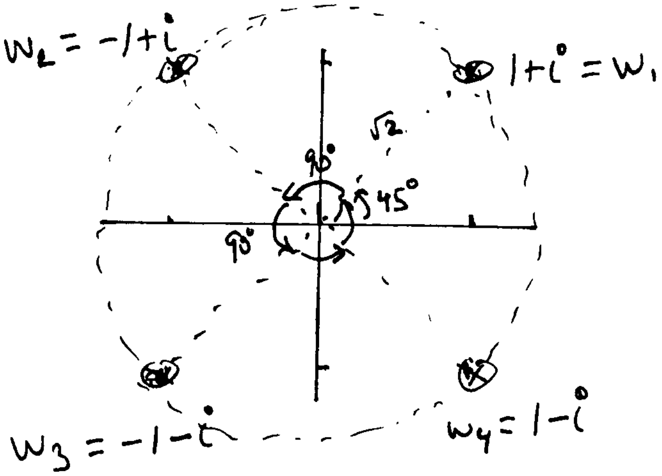
$\frac{1080°}{4} = \frac{3 \cdot 360°}{4} = 3 \cdot 90°$

$$w_5 = \sqrt{2}[\cos(45°+360°) + i \sin(45°+360°)] = w_1$$

$\frac{4 \cdot 360°}{4}$

Not new.

ex (cont'd) What are the standard forms of these four 4th roots of -4?



$w_1 = 1+i$

$w_2 = -1+i$

$w_3 = -1-i$

$w_4 = 1-i$

ex: Suppose that $P(x) = x^4+4$.
What are the zeros of $P(x)$? Solve $x^4+4=0$

Solve $x^4=-4$.

That is, find the 4th roots of -4.

Answer: $1+i, -1+i, -1-i, 1-i$.

ex: How does $P(x) = x^4+4$ factor into linear factors over the complex numbers?

linear factors: $(x-1-i)\,(x+1-i)\,(x+1+i)\,(x-1+i)$

zeros: $1+i \qquad -1+i \qquad -1-i \qquad 1-i$

ex: [Pop quiz, might have been] Factor x^4+4 over the reals.

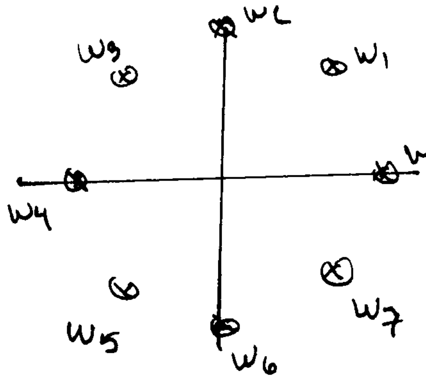
$$P(x) = \big[(x-1-i)(x-1+i)\big]\cdot\big[(x+1-i)(x+1+i)\big]$$

$$= (x^2-2x+2)\,(x^2+2x+2)$$

Remark: $\boxed{(x-\alpha-\beta i)\,(x-\alpha+\beta i) = x^2-2\alpha+\alpha^2+\beta^2}$

ex: What are the 8^{th} roots of 1 ?

$$1 = 1 \cdot (\cos 0^\circ + i \sin 0^\circ)$$



$w_0 = 1$

$w_1 = \cos 45^\circ + i \sin 45^\circ = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$

$w_2 = i$

$w_2 = -\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$

$w_4 = -1$

$w_5 = -\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$

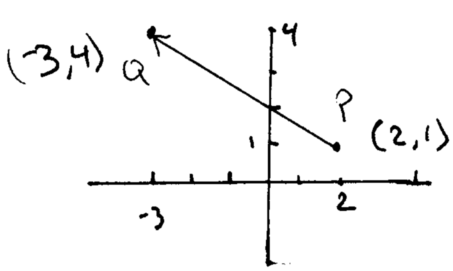
$w_6 = -i$

$w_7 = \frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$

[We will probably have to skip section 8.4: Parametric Curves]

9.1 Vectors in Two dimensions

(1)



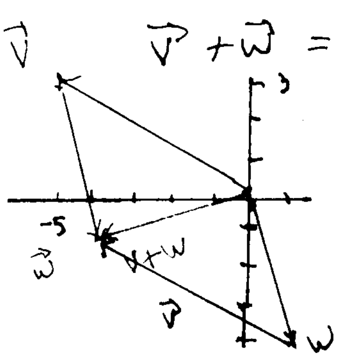
$\vec{v} = \overrightarrow{PQ} = \langle (-3)-2, 4-1 \rangle = \langle -5, 3 \rangle$

↑ "component form" of a vector.

<u>Addition</u> using component form

$\vec{v} = \langle -5, 3 \rangle \qquad \vec{w} = \langle 1, -4 \rangle$

$\vec{v} + \vec{w} = \langle -5+1, 3+(-4) \rangle = \langle -4, -1 \rangle$



Scalar multiplication using component form

$\vec{v} = \langle -5, 3 \rangle \qquad 3\vec{v} = 3\langle -5, 3 \rangle$
$= \langle -15, 9 \rangle$

$-\frac{1}{2}\vec{v} = -\frac{1}{2}\langle -5, 3 \rangle = \langle \frac{5}{2}, -\frac{3}{2} \rangle = \langle 2.5, -1.5 \rangle$

Combined

$\vec{v} = \langle -5, 3 \rangle \qquad \vec{w} = \langle 1, -4 \rangle$

$10\vec{v} - 2\vec{w} = 10\langle -5, 3 \rangle - 2\langle 1, -4 \rangle$
$= \langle -50, 30 \rangle + \langle -2, 8 \rangle$
$= \langle -52, 38 \rangle$

Defn: If $\vec{v} = \langle a, b \rangle$ then the length or magnitude is $|\vec{v}| = \sqrt{a^2 + b^2}$

ex: If $\vec{v} = \langle -5, 3 \rangle$, the length $|\vec{v}| = \sqrt{(-5)^2 + 3^2}$
$= \sqrt{34}$ = almost 6
$= 5.83$

ex [of the property $|c\vec{v}| = |c|\,|\vec{v}|$]

Suppose $c = -10$ and $\vec{v} = \langle 3, -4 \rangle$; then $|\vec{v}| = \sqrt{3^2 + (-4)^2} = 5$

$c\vec{v} = -10\langle 3, -4 \rangle = \langle -30, 40 \rangle$

Observe: $|c\vec{v}| = \sqrt{(-30)^2 + 40^2} = 50$
$|c|\,|\vec{v}| = |-10| \cdot 5 = 10 \cdot 5$

NOTATION: $\vec{i} = \langle 1, 0 \rangle \qquad \vec{j} = \langle 0, 1 \rangle$ by definition of $\vec{i}$ and $\vec{j}$

$\vec{v} = \langle 3, -4 \rangle = \langle 3, 0 \rangle + \langle 0, -4 \rangle = 3\langle 1, 0 \rangle - 4\langle 0, 1 \rangle$
$= 3\vec{i} - 4\vec{j}$

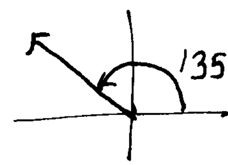
ex: [Rectangular vs. Polar form of a vector]

Component form of vector $= \langle a, b \rangle = \vec{v}$

Form that expresses <u>direction</u> and <u>magnitude</u>:

$$\vec{v} = |\vec{v}| \langle \cos\theta, \sin\theta \rangle$$

<u>exmple</u>: $\vec{v} = \langle -5, 5 \rangle$



$$|\vec{v}| = \sqrt{(-5)^2 + 5^2} = \sqrt{50} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$$

$$\tan\theta = \frac{b}{a} = \frac{5}{-5} = -1 \Rightarrow \theta = 135^\circ$$

<u>polar</u> <u>form</u> of the vector [<u>NOT</u> in the textbook]

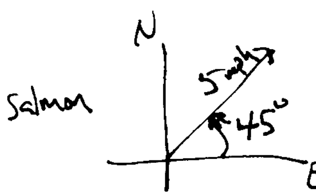
$$\vec{v} = \boxed{\langle -5, 5 \rangle} \quad \text{Rectangular form}$$

$$= 5\sqrt{2} \left\langle \frac{-5}{5\sqrt{2}}, \frac{5}{5\sqrt{2}} \right\rangle$$

$$= 5\sqrt{2} \left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle = 5\sqrt{2} \left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$$

$$= \boxed{5\sqrt{2} \langle \cos 135^\circ, \sin 135^\circ \rangle} \quad \text{polar form}$$

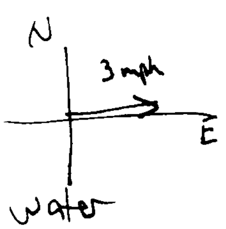
58) Salmon velocity in still water? Direction = N 45° E
Magnitude = 5 mi/hr



$$\vec{v} = 5 \langle \cos 45^\circ, \sin 45^\circ \rangle$$

$$= 5 \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle = \left\langle \frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2} \right\rangle$$

water velocity?



$$\vec{w} = \langle 3, 0 \rangle$$

Net motion? $\vec{v} + \vec{w} = \left\langle \frac{5\sqrt{2}}{2} + 3, \frac{5\sqrt{2}}{2} \right\rangle$

What about direction and magnitude? [Out of time.]

Added after class - to finish up the last problem.

what is the speed (i.e. the magnitude of the velocity vector) of the salmon swimming with the current?

$$|\vec{v}+\vec{w}| = \left|\left\langle \frac{5\sqrt{2}}{2}+3,\ \frac{5\sqrt{2}}{2}\right\rangle\right| = |\langle 6.536,\ 3.536\rangle|$$

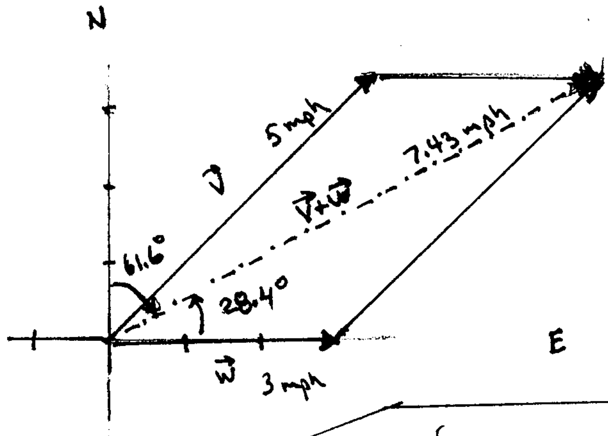
$$= \sqrt{6.536^2+3.536^2} = \boxed{7.43 \text{ miles/hr}}$$

Direction of the salmon swimming with the current?

$$\tan\theta = \frac{3.536}{6.536} = 0.54097 \qquad \text{so}$$

$$\theta = \tan^{-1} 0.54097 = 28.4^\circ \quad \text{north of due east}$$

or $\boxed{N\ 61.6^\circ\ E}$



Summary of contexts where the idea of rectangular vs. polar appears. (via an example)

	Rectangular	Polar	Notes:
coordinates in the plane	$(x,y)=(-5,5)$	$(r,\theta)=(5\sqrt{2},\ 135^\circ)$	Points (locations) can't be added
complex numbers	$-5+5i$	$5\sqrt{2}(\cos 135^\circ + i\sin 135^\circ)$	complex numbers can be added and multiplied.
vectors in the plane	$\langle -5, 5\rangle$	$5\sqrt{2}\langle\cos 135^\circ, \sin 135^\circ\rangle$	Vectors can be added.