Remark [actually elaboration on a remark from last time.]

If r_1 and r_2 are zeros of a polynomial $P(x)$, then $(x-r_1)(x-r_2) = x^2-(r_1+r_2)x+r_1r_2$ will be a quadratic factor of $P(x)$. In particular, if r_1 and r_2 are complex conjugates, say $r_1 = \alpha+\beta i$ and $r_2 = \alpha-\beta i$, then

$r_1+r_2 = 2\alpha$ and $r_1r_2 = \alpha^2-\beta^2 i^2 = \alpha^2+\beta^2$.

The corresponding irreducible quadratic factor of $P(x)$ is then

$$x^2-(r_1+r_2)x+r_1r_2 = x^2-2\alpha x+(\alpha^2+\beta^2)$$

Example: If $r_1 = 2+3i$ and $r_2 = 2-3i$ then

$r_1+r_2 = 4$ and $r_1r_2 = 13$.

Then the quadratic factor with these zeros is

$$x^2-4x+13.$$

[At this point we engaged in Group Quiz #8 and discovered that x^6+27, when factored over the real numbers, factors as:

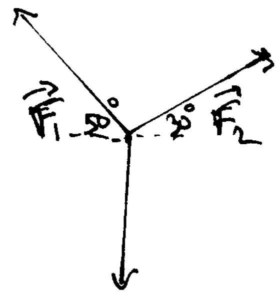
$$x^6+27 = (x^2-3x+3)(x^2+3)(x^2+3x+3)$$]

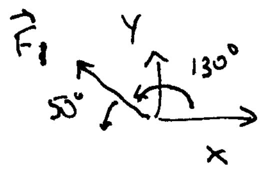
9.1

73) A weight of 100 lbs is supported by two cables at the given angles. What is the tension on each cable?

$|\vec{F_1}| = T_1$

$|F_2| = T_2$





$$\vec{w} = \langle 0, -100 \rangle = -100\,\vec{j}$$

$$\vec{F_2} = T_2 \langle \cos 30°, \sin 30° \rangle = T_2 (\cos 30°\,\vec{i} + \sin 30°\,\vec{j})$$

$$\vec{F_1} = T_1 \langle \cos 130°, \sin 130° \rangle = T_1 (\cos 130°\,\vec{i} + \sin 130°\,\vec{j})$$

The sum of the vectors is $\vec{0}$ because the knot is not moving (equilibrium).

$$\vec{F_1} + \vec{F_2} + \vec{w} = \vec{0} \Rightarrow \vec{F_1} + \vec{F_2} = -\vec{w}$$

$$\langle T_1 \cos 130°, T_1 \sin 130° \rangle + \langle T_2 \cos 30°, T_2 \sin 30° \rangle = \langle 0, 100 \rangle$$

Equivalent System of equations:

$$\begin{cases} T_1 \cos 130° + T_2 \cos 30° = 0 \\ T_1 \sin 130° + T_2 \sin 30° = 100 \end{cases}$$

$$\begin{cases} -0.643\, T_1 + 0.866\, T_2 = 0 \\ 0.766\, T_1 + 0.5\, T_2 = 100 \end{cases}$$

By Use of a TI84:

$$\begin{bmatrix} \cos 130° & \cos 30° & 0 \\ \sin 130° & \sin 30° & 100 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 87.94 \\ 0 & 1 & 65.27 \end{bmatrix}$$

[See Chap 10 specifically Section 10.3]

Answer: $\boxed{T_1 = 87.94 \text{ lbs}}$ $\boxed{T_2 = 65.27 \text{ lbs}}$

Written after class: we can solve the system without use of a TI-84:

$$① \quad T_1 \cos 130° + T_2 \cos 30° = 0$$

$$② \quad T_1 \sin 130° + T_2 \sin 30° = 100$$

To eliminate T_2, multiply ① by $-\sin 30°$ and ② by $\cos 30°$, then add.

$$-\sin 30° \cdot ① : \quad -T_1 \sin 30° \cos 130° - T_2 \sin 30° \cos 30° = 0$$

$$\cos 30° \cdot ② : \quad T_1 \cos 30° \sin 130° + T_2 \sin 30° \cos 30° = 100 \cos 30°$$

$$T_1 (\sin 130° \cos 30° - \cos 130° \sin 30°) = 100 \cos 30°$$

or $$T_1 \sin(130° - 30°) = T_1 \sin 100° = 100 \cos 30°$$

$$\therefore \quad T_1 = \frac{100 \cos 30°}{\sin 100°} = \boxed{87.94 \text{ lbs.}}$$

To eliminate T_1:

$$-\sin 130° \cdot ① : \quad -T_1 \sin 130° \cos 130° - T_2 \sin 130° \cos 30° = 0$$

$$\cos 130° \cdot ② : \quad T_1 \sin 130° \cos 130° + T_2 \cos 130° \sin 30° = 100 \cos 130°$$

$$T_2 (\sin 30° \cos 130° - \cos 30° \sin 130°) = 100 \cos 130°$$

OR $$T_2 \sin(30° - 130°) = T_2 \sin(-100°) = -T_2 \sin 100° = 100 \cos 130°$$

$$\therefore \quad T_2 = \frac{100 \cos 130°}{-\sin 100°} = \boxed{65.27 \text{ lbs}}$$