10.6 Determinants

Defn: If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then the determinant of A

$$\det(A) = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = (a)(d) - (b)(c).$$

ex: $A = \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}$ $\det(A) = (1)(7) - (3)(5)$
$= 7 - 15 = -8$

$B = \begin{bmatrix} -5 & \frac{1}{2} \\ \frac{3}{4} & \frac{5}{2} \end{bmatrix}$ $\det(B) = (-5)(\frac{5}{2}) - (\frac{1}{2})(\frac{3}{4})$
$= -\frac{25}{2} - \frac{3}{8} = -\frac{100}{8} - \frac{3}{8}$
$= \frac{-103}{8}$

Defn of 3×3 Determinant [see textbook]

10.6 19) $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & -2 & 4 \\ 0 & 1 & -3 \end{bmatrix}$

$$\det(A) = 2\begin{vmatrix} -2 & 4 \\ 1 & -3 \end{vmatrix} - 1\begin{vmatrix} 0 & 4 \\ 0 & -3 \end{vmatrix} + 0\begin{vmatrix} 0 & -2 \\ 0 & 1 \end{vmatrix}$$

$$= 2[(-2)(-3) - (4)(1)] - 1[\underbrace{0(-3) - 4(0)}_{0}] + 0[\overset{=0}{\text{whatever}}]$$

$$= 2[6 - 4] = 2[2] = 4$$

Remark: This is an example of "expanding the determinant along the first row."

You may expand along any row or any column.

ex: $\det(A) = 2\begin{vmatrix} -2 & 4 \\ 1 & -3 \end{vmatrix} - 0\begin{vmatrix} 1 & 0 \\ 1 & -3 \end{vmatrix} + 0\begin{vmatrix} 1 & 0 \\ -2 & 4 \end{vmatrix} = 2(6-4)$
$= 4$

ex: $B = \begin{bmatrix} 5 & 0 & 0 \\ 71 & -2 & 0 \\ 169 & -\sqrt{2} & 3 \end{bmatrix}$

$$\det(B) = 5 \begin{vmatrix} -2 & 0 \\ -\sqrt{2} & 3 \end{vmatrix} - 0 \cdot \begin{vmatrix} 71 & 0 \\ 169 & 3 \end{vmatrix} + 0 \cdot \begin{vmatrix} 71 & -2 \\ 169 & -\sqrt{2} \end{vmatrix}$$

$$= 5\left[(-2)(3) - (0)(-\sqrt{2})\right] = (5)(-2)(3) = \boxed{-30}$$

Facts: (1) An $n \times n$ matrix A is invertible if and only if $\det(A) \neq 0$.

ex: $A = \begin{bmatrix} 3 & 6 \\ 15 & 30 \end{bmatrix}$ Does A^{-1} exist?

$\det(A) = (3)(30) - (6)(15)$
$= 90 - 90 = 0$ No!

"A is a singular matrix."

(2) If A and B are $n \times n$ matrices then

$$\det(AB) = \det(A) \cdot \det(B)$$

Find $\det(A)$:

23) $A = \begin{bmatrix} 1 & 3 & 7 \\ 2 & 0 & 8 \\ 0 & 2 & 2 \end{bmatrix}$ $\quad -2R_1 + R_2 \to R_2 \begin{bmatrix} 1 & 3 & 7 \\ 0 & -6 & -6 \\ 0 & 2 & 2 \end{bmatrix}$

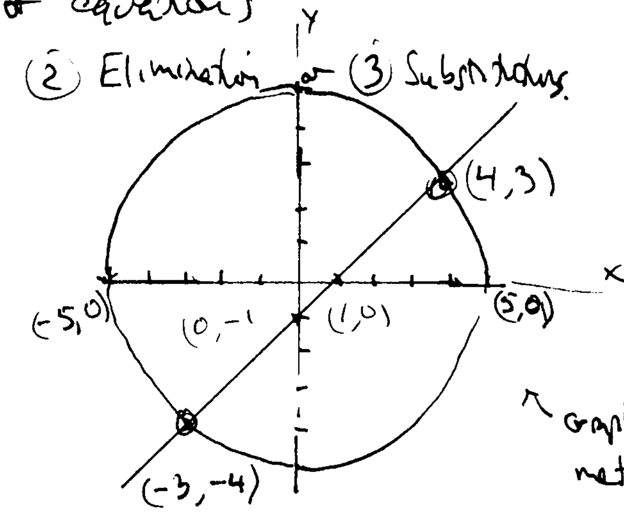
$\frac{1}{3}R_2 + R_3 \to R_3 \begin{bmatrix} 1 & 3 & 7 \\ 0 & -6 & -6 \\ 0 & 0 & 0 \end{bmatrix} = B$

$$\det(A) = \det(B) = 0 \begin{vmatrix} 3 & 7 \\ -6 & -6 \end{vmatrix} - 0 \begin{vmatrix} 1 & 7 \\ 0 & -6 \end{vmatrix} + 0 \begin{vmatrix} 1 & 3 \\ 0 & -6 \end{vmatrix} = \boxed{0}$$

10.8 Nonlinear systems of equations

Methods: (1) Graphing (2) Elimination or (3) Substitution.

ex. (1) $\begin{cases} x^2+y^2=25 \\ x-y=1 \end{cases}$ (2)



↖ Graphing method

By substitution

Solve (2) for x: $x=y+1$

Sub into (1):
$$(y+1)^2+y^2=25$$
$$y^2+2y+1+y^2=25$$
$$2y^2+2y+1-25=0$$
$$2y^2+2y-24=0$$
$$2(y^2+y-12)=0$$
$$2(y-3)(y+4)=0$$

case 1 → $y=3$ or $y=-4$ ← case 2

$x=3+1$
$=4$
$(x,y)=\boxed{(4,3)}$

$x=-4+1$
$=-3$
$(x,y)=\boxed{(-3,-4)}$

10.8 18) Solve $\begin{cases} x^2+y^2=4x \\ x=y^2 \end{cases}$

Try elimination:
$$\begin{array}{rcl} x^2+y^2 &=& 4x \\ -y^2 &=& -x \\ \hline x^2 &=& 3x \end{array}$$
$$x^2-3x=0$$
$$(x-3)x=0$$
$$x=3 \quad \text{or} \quad x=0$$

Back-substitute:

$y^2=3$

$y=\pm\sqrt{3}$

$(x,y)=\boxed{(3,\sqrt{3})}$

or

$(x,y)=\boxed{(3,-\sqrt{3})}$

$\approx(3,-1.73)$

$0=y^2$

$y=\pm\sqrt{0}=0$

$(x,y)=\boxed{(0,0)}$

20) ① ② $\begin{cases} x^4+y^3=17 \\ 3x^4+5y^3=53 \end{cases}$

$-3\cdot$①: $-3x^4-3y^3=-51$

②: $3x^4+5y^3=53$

$2y^3=2$

$y^3=1$

$0=y^3-1=(y-1)(y^2+y+1)$

$y=1$ complex solutions

Back substitute:

$x^4+1=17$

$x^4=16$

$0=x^4-16=(x^2-4)(x^2+4)$

$=(x-2)(x+2)(x^2+4)$ complex

real solns: $x=2$, $x=-2$

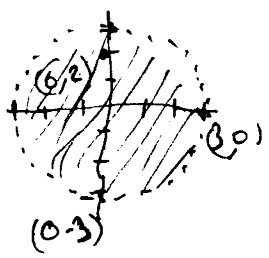
$(x,y)=\boxed{(2,1)}$ or $\boxed{(-2,1)}$

M141 5/11/15

10.9 Non linear equations in two variables (needed in Calculus III and double integrals)

ex: Graph $x^2+y^2<9$

Step 1 graph $x^2+y^2=9$



Step 2 Take a test point, like $(x,y)=(0,0)$.

Does $(0,0)$ satisfy $x^2+y^2<9$?

$0^2+0^2<9$ is TRUE.

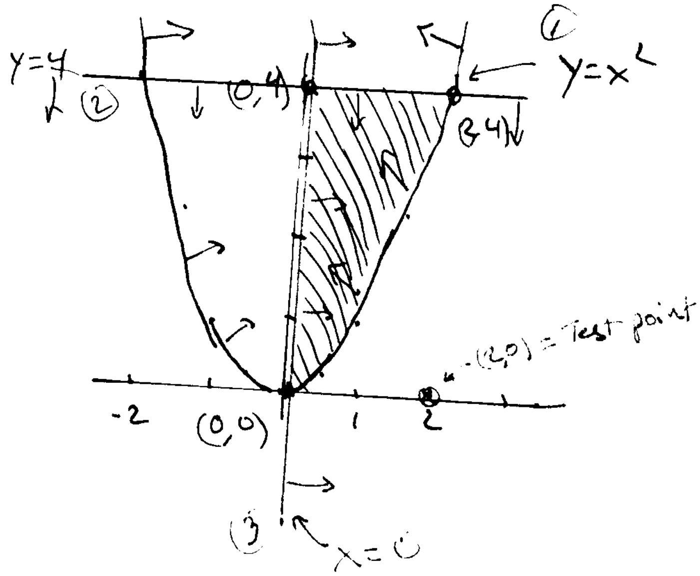
Shade the inside of the circle.

NOTE: $(x,y)=(3,0)$ is not a solution.

10.9 30) ① $y \geq x^2$

② $y \leq 4$

③ $x \geq 0$

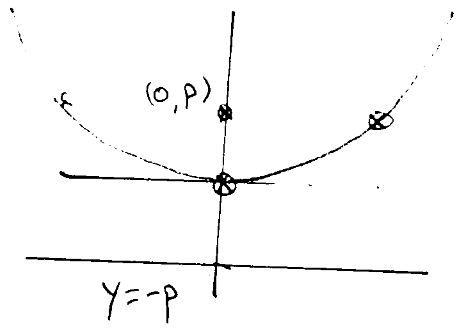


Step 2 Does (2,0) satisfy ①? No.

Does (2,0) satisfy ②? Yes

Does (2,0) satisfy ③? Yes.

11.1 Parabolas



$\boxed{x^2 = 4py}$

where p = vertex-to-focus (directed) distance

ex: Focus $(0,2)$, directrix: $y=-2$

Find the equation:

Since $p=2$ the equation is $x^2 = 4(2)y$

$\boxed{x^2 = 8y}$