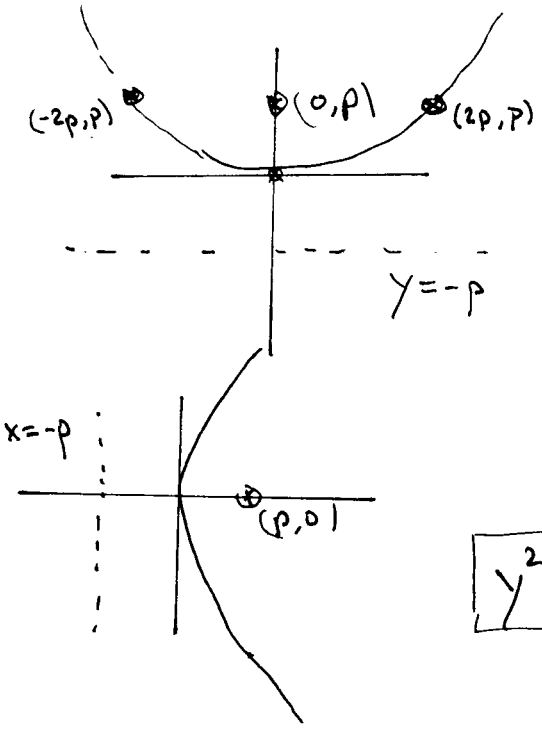
11.1 Parabolas



$$x^2 = 4py$$

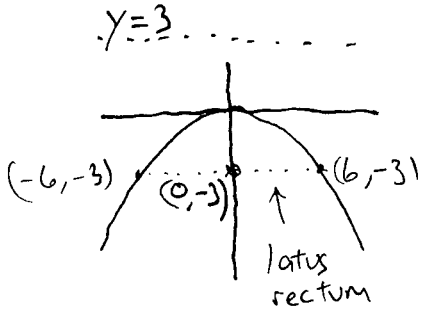
or $y = \frac{1}{4p}x^2$

or $y = ax^2$ with $a = \frac{1}{4p}$

NOTE: p may be positive or negative.

$$y^2 = 4px$$

44) Find the equation:



$p = -3$ so

$x^2 = 4(-3)y$

$x^2 = -12y$

check: if $(x, y) = (6, -3)$

$6^2 \overset{\checkmark}{=} -12(-3)$

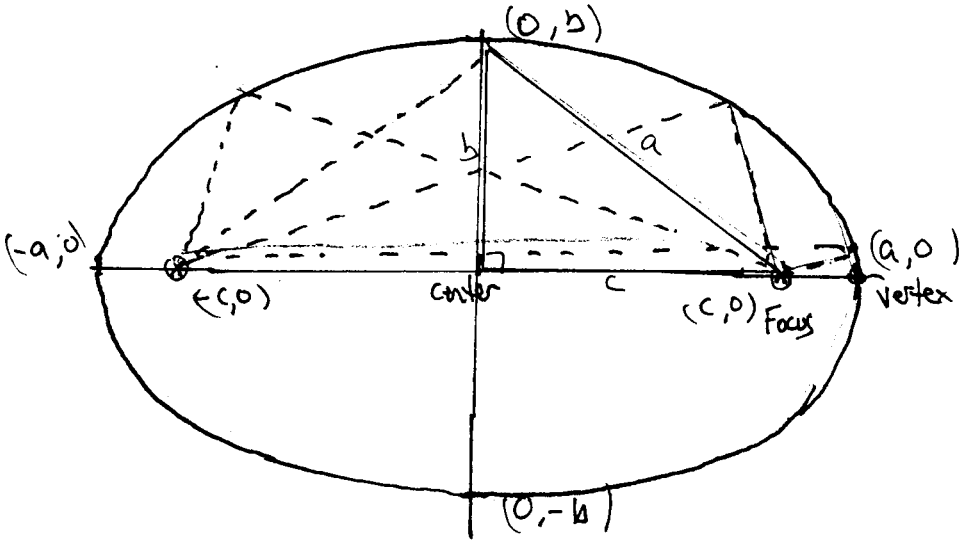
NOTE:

$|4p| = |-12| = 12$

= "Focal diameter"

= length of latus rectum

11.2 Ellipses



$2a$ = length of "string"
= length of major axis

a = length of semimajor axis
= length of the hypotenuse in the red triangle

$$\boxed{a^2 = b^2 + c^2}$$

where
c = center-to-focus distance
b = length of semiminor axis

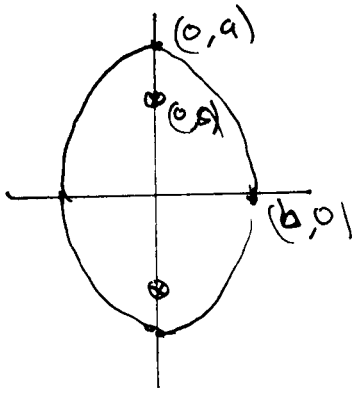
$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

$$\boxed{e = \text{eccentricity} = \frac{c}{a}}$$

Remark: If you know any two of a, b, c or e, you can calculate the other two.

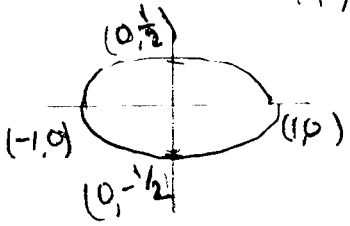
Remark: For the earth's orbit $a = 1$ astronomical unit $= 1$ a.u.

eccentricity $= .0167 = e$



$$\boxed{\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1}$$

17) $x^2 + 4y^2 = 1 \Rightarrow \dfrac{x^2}{1^2} + \dfrac{y^2}{(1/2)^2} = 1$



So $a = 1$

$b = 1/2$

$c^2 = a^2 - b^2 = 1 - \frac{1}{4} = \frac{3}{4} \Rightarrow c = \frac{\sqrt{3}}{2}$

$e = \dfrac{c}{a} = \dfrac{\sqrt{3}/2}{1} = .866$

ex: $\frac{x^2}{25} + \frac{y^2}{9} = 1$ Find the x- and y-intercepts.

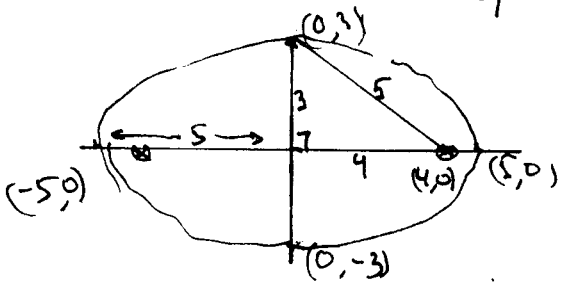
x-intercept? Set $y=0$, Solve for x:

$\frac{x^2}{25} + 0 = 1 \Rightarrow x^2 = 25$

$x = \pm\sqrt{25} = \pm 5$

y-intercept? Set $x=0$

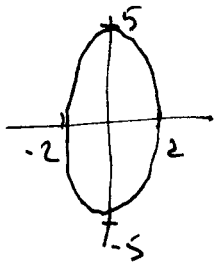
$\frac{y^2}{9} = 1 \Rightarrow y^2 = 9 \Rightarrow y = \pm 3$



$a = 5$ $\quad e = \frac{c}{a} = \frac{4}{5} = 0.8$

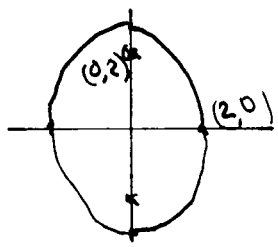
$b = 3$

$c = \sqrt{5^2 - 3^2} = \sqrt{16} = 4$

24) 

What is the equation of this ellipse?

$\frac{x^2}{4} + \frac{y^2}{25} = 1$

25) 

$b = 2$

$c = 2$

$a^2 = b^2 + c^2 = 2^2 + 2^2 = 8$

$a = \sqrt{8} = 2\sqrt{2} \approx 1.4$

$\frac{x^2}{4} + \frac{y^2}{8} = 1$

12) $\frac{4x^2}{100} + \frac{25y^2}{100} = \frac{100}{100}$

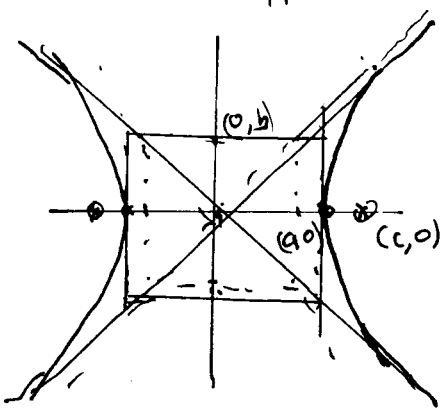
$\frac{x^2}{25} + \frac{y^2}{4} = 1$

what are a, b, c and e?

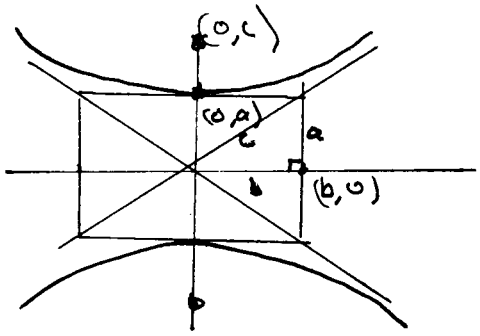
$a = 5$ $\quad c^2 = a^2 - b^2 = 25 - 4 = 21 \Rightarrow c = \ldots \sqrt{21}$

$b = 2$ $\quad e = \frac{c}{a} = \frac{\sqrt{21}}{5} \approx .92$

10.3 Hyperbolas



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

a = center-to-vertex distance

c = center-to-focus distance

$e = \frac{c}{a}$ = eccentricity

b = "other" dimension of the central box.

$c^2 = a^2 + b^2$

Equations of asymptotes:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$$

Eqn of asymptotes.

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 0$$

Remarks: (1) If you know two of a, b, c, e you know all four.

(2) It's no longer the case that $a > b$. You may have $a > b$, $a = b$, or $a < b$. But $c > a$ and $c > b$.

(3) a^2 is under the positive variable.

10) $\frac{y^2}{9} - \frac{x^2}{16} = 1$

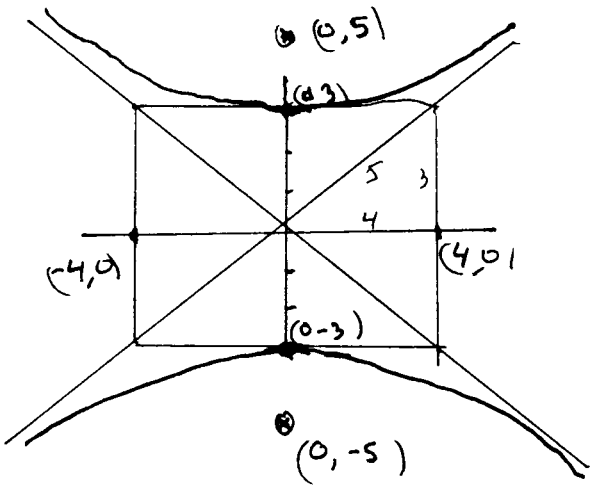
Find the y-intercepts: Solve $\frac{y^2}{9} = 1$

So $y^2 = 9$ and $y = \pm 3$.

Are there x-intercepts? Set $y = 0$:

$-\frac{x^2}{16} = 1 \Rightarrow x^2 = -16$ has no real solutions

$a = 3$ $\quad b = 4$



$a = 3$

$b = 4$

$c^2 = 3^2 + 4^2 = 25$

$c = 5$

Vertices: $(0, \pm 3)$

Foci: $(0, \pm 5)$

$e = \frac{c}{a} = \frac{5}{3} = 1.667$

Equations of the asymptotes:

$$\frac{y^2}{9} - \frac{x^2}{16} = 0$$

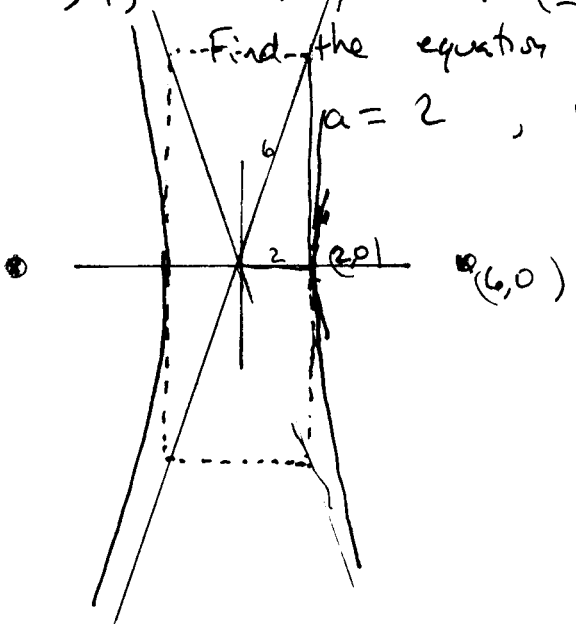
$$\frac{y^2}{9} = \frac{x^2}{16} \Rightarrow y^2 = \frac{9}{16}x^2 \Rightarrow \boxed{y = \pm\frac{3}{4}x}$$

That is, $y = \frac{3}{4}x$ or $y = -\frac{3}{4}x$

11.3

34) Given: Foci: $(\pm 6, 0)$ Vertices: $(\pm 2, 0)$

Find the equation of the hyperbola.



$a = 2$, $c = 6$

$b^2 = c^2 - a^2$

$= 36 - 4 = 32$

$b = \sqrt{32} = 4\sqrt{2}$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ becomes

$$\boxed{\frac{x^2}{4} - \frac{y^2}{32} = 1}$$

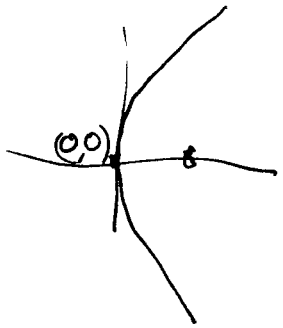
Note: $\frac{c}{a} = 3 = e$

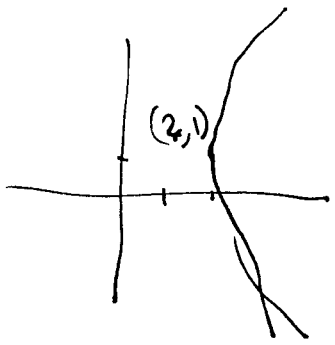
Intro to 11.4 Shifted conics

Main idea: Replacing x with $x-2$ and y with $y-1$ shifts the position, but does not alter the shape or size.

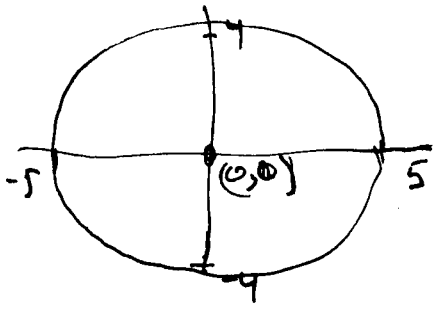
Shifted to $(x,y)=(2,1)$

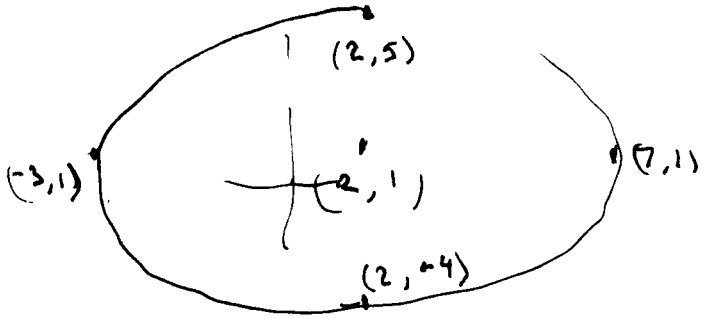
ex: (parabola) $y^2 = 8x$ $\qquad$ $(y-1)^2 = 8(x-2)$





ex: (ellipse) $\frac{x^2}{25} + \frac{y^2}{16} = 1$ $\qquad$ $\frac{(x-2)^2}{25} + \frac{(y-1)^2}{16} = 1$





ex: (hyperbola) $\frac{x^2}{4} - \frac{y^2}{4} = 1$ $\qquad$ $\frac{(x-2)^2}{4} - \frac{(y-1)^2}{4} = 1$