11.4 Shifted conics

Graph $\longrightarrow$ Standard equation $\longleftarrow$ Non standard equation

Standard equation $\longrightarrow$ Graph

ex: Graph $4x^2+9y^2=36$

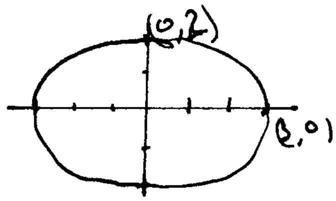
$$\frac{4x^2}{36}+\frac{9y^2}{36}=\frac{36}{36} \Rightarrow \frac{x^2}{9}+\frac{y^2}{4}=1$$

This is an ellipse, center $=(0,0)$

$a=3 \quad b=2$

$c=\sqrt{a^2-b^2}=\sqrt{9-4}=\sqrt{5}$

$e=\frac{c}{a}=\frac{\sqrt{5}}{3}$



ex: 6) Graph $\frac{(x-3)^2}{16}+(y+3)^2=1$

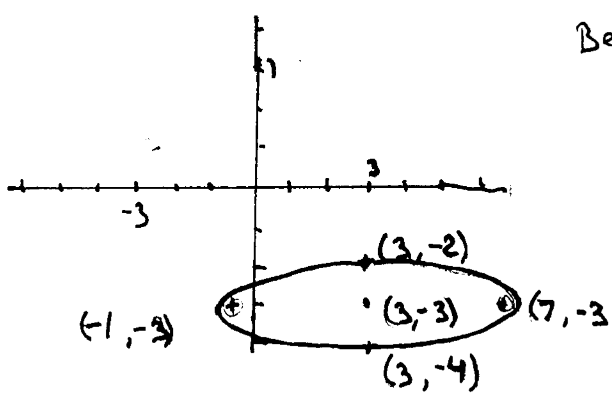
center $=(3,-3)$

$a=4$

$b=1$

Because $a^2=b^2+c^2$, $c=\sqrt{a^2-b^2}=\sqrt{15}\approx 3.9$

so $e=\frac{c}{a}=\frac{\sqrt{15}}{4}=0.968$



vertices $=(-1,-3),(7,-3)$

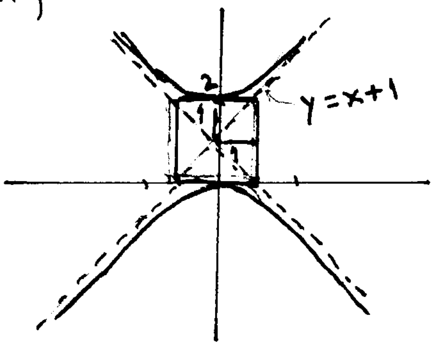
foci $=(3\pm\sqrt{15},-3)$

$\approx(6.87,-3),(-0.87,-3)$

major axis $=2a=8$

minor axis $=2b=2$

11.4 21) Find an equation of this hyperbola:



a = center-to-vertex distance $= 1$

Slope ~~of~~ asymptote $= 1 = \frac{b}{a}$

So $b = 1$

Note
$(h,k) = (0,1)$
$a = 1$
$b = 1$

If the center were at $(0,0)$ the equation would be $\frac{y^2}{1} - \frac{x^2}{1} = 1$

But since the center is at $(0,1)$ our equation is

$$\frac{(y-1)^2}{1} - \frac{x^2}{1} = 1 \quad \text{or} \quad (y-1)^2 - x^2 = 1$$

24) Graph

$$9x^2 - 36x + 4y^2 = 0$$

$$9(x^2 - 4x + 4) + 4y^2 = 0 + 36$$

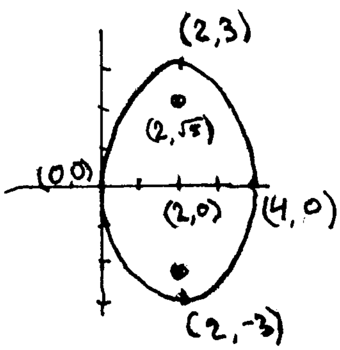
$$9(x-2)^2 + 4y^2 = 36$$

$$\frac{9(x-2)^2}{36} + \frac{4y^2}{36} = \frac{36}{36}$$

$$\frac{(x-2)^2}{4} + \frac{y^2}{9} = 1$$

center $= (h,k) = (2,0)$
$a = 3$ = length of semimajor axis
$b = 2$ = " minor "

$c = \sqrt{a^2 - b^2}$
$= \sqrt{9-4} = \sqrt{5}$



center $= (2,0)$
vertices $= (2,3), (2,-3)$
foci $= (2,\sqrt{5}), (2,-\sqrt{5})$

11.4 30) $4x^2 - 4x - 8y + 9 = 0$ will be a parabola (no y^2 term)

Std eqn for parabola:

$y^2 = 4px$ OR $(y-k)^2 = 4p(x-h)$

or $x^2 = 4py$ or $(x-h)^2 = 4p(y-k)$

$$4x^2 - 4x = 8y - 9$$

$$4\left(x^2 - x + \tfrac{1}{4}\right) = 8y - 9 + 1$$

$$4\left(x - \tfrac{1}{2}\right)^2 = 8y - 8$$

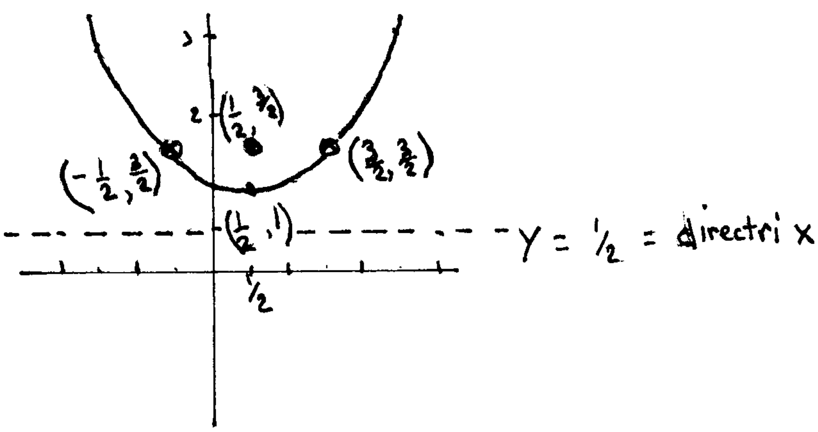
Divide by 4: $\left(x - \tfrac{1}{2}\right)^2 = 2y - 2$

$$\left(x - \tfrac{1}{2}\right)^2 = 4 \cdot \tfrac{1}{4}(2y - 2)$$

$$\left(x - \tfrac{1}{2}\right)^2 = 4 \cdot \tfrac{1}{2}(y - 1)$$

vertex $= (h, k)$
$= \left(\tfrac{1}{2}, 1\right)$

and $p = \tfrac{1}{2}$ = vertex-to-focus distance

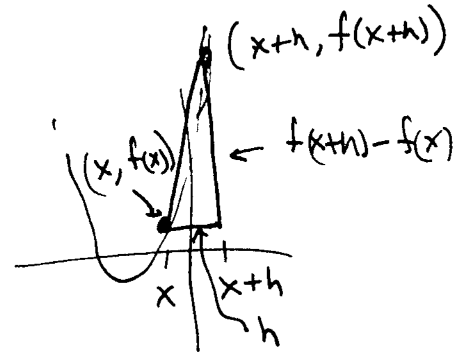


focus $= \left(\tfrac{1}{2}, \tfrac{3}{2}\right)$

Some review

2.4 $f(x) = x^2 + 3x + 2$

$$\frac{f(x+h) - f(x)}{h}$$



Step 1: $f(x+h) = (x+h)^2 + 3(x+h) + 2$

$= x^2 + 2hx + h^2 + 3x + 3h + 2$

Step 2: $f(x+h) - f(x) = x^2 + 2hx + h^2 + 3x + 3h + 2$
$\quad - x^2 \qquad - 3x \qquad - 2$

$= 2hx + h^2 + 3h$

$= h(2x + h + 3)$

Step 3: $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(2x+h+3)}{h} = 2x + h + 3$

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45) $P(x) = 2x^4 + x^3 + 2x^2 - 3x - 2$

- Find all real zeros
- Sketch

\# pos. real zeros: 1 variation of sign $\Rightarrow$ 1 positive zero

$P(-x) = 2x^4 - x^3 + 2x^2 + 3x - 2$

\# neg. real zeros: 3 variations $\Rightarrow$ 3 or 1 negative zero.

Rational zeros theorem: $p: \pm 1 \quad \pm 2$
$q: \pm 1 \quad \pm 2$ $\Rightarrow \frac{p}{q}: \pm 1, 2, \frac{1}{2}$

Is 1 a zero?

$$
\begin{array}{r|rrrrr}
1 & 2 & 1 & 2 & -3 & -2 \\
 & & 2 & 3 & 5 & 2 \\
\hline
-1/2 & 2 & 3 & 5 & 2 & \vert\ 0 = P(1) \\
 & & -1 & -1 & -2 & \\
\hline
 & 2 & 2 & 4 & \vert\ 0 &
\end{array}
$$

Yes

$P(x) = (x-1)(2x^3+3x^2+5x+2)$

$= (x-1)(x+\tfrac{1}{2})(2x^2+2x+4)$

$= (x-1)(x+\tfrac{1}{2})\,2(x^2+x+2)$

$= (x-1)(2x+1)(x^2+x+2)$

zeros: 1 $\quad -1/2 \quad$? $\quad$?

For the other two zeros:

Solve

$x^2+x+2=0$

$x^2+x+\frac{1}{4} = -2+\frac{1}{4}$

$(x+\frac{1}{2})^2 = -\frac{7}{4}$

$x+\frac{1}{2} = \pm\sqrt{\frac{-7}{4}} = \pm\frac{i\sqrt{7}}{2}$

$x = -\frac{1}{2} \pm \frac{i\sqrt{2}}{2}$ so no more real zeros (so no more x-intercepts)

Some topics we never got to (will NOT appear on the final)

10.7 Partial fractions decomposition

16) Find the partial fraction decomp. of

$$\frac{x+6}{x(x+3)} = \frac{A}{x} + \frac{B}{x+3}$$

Task: Find the constants A and B

Multiply by LCD $= x(x+3)$:

$$x+6 = A\frac{\cancel{x}(x+3)}{\cancel{x}} + \frac{Bx\cancel{(x+3)}}{\cancel{x+3}}$$

$$x+6 = A(x+3) + Bx$$

If these are the same function they should evaluate the same.

Set $x=0$: $\quad 6 = 3A \Rightarrow A=2$

Set $x=-3$: $\quad 3 = -3B \Rightarrow B=-1$

Answer: $\dfrac{x+6}{x(x+3)} = \dfrac{2}{x} + \dfrac{-1}{x+3}$