

10.7 Partial fraction decomposition

$$18) \quad \frac{x-12}{x^2-4x} = \frac{x-12}{x(x-4)}$$

$$\frac{x-12}{x(x-4)} = \frac{A}{x} + \frac{B}{x-4}$$

Multiply by
the LCD
 $x(x-4)$

$$x-12 = \frac{A \cancel{x(x-4)}}{x} + \frac{B x \cancel{(x-4)}}{x-4}$$

$$x-12 = A(x-4) + Bx$$

← An equation
of two degree-1
polynomials

$$x-12 = Ax - 4A + Bx$$

$$x-12 = (A+B)x - 4A$$

← Combine like terms

These can be equal polynomials only if

$$(1) \quad \begin{cases} A+B = 1 \end{cases}$$

← Leading coeffs are equal

$$(2) \quad \begin{cases} -4A = -12 \end{cases}$$

← constant " " "

$$\boxed{A = +3}$$

$$3 + B = 1$$

$$\boxed{B = -2}$$

$$\boxed{\frac{x-12}{x(x-4)} = \frac{3}{x} - \frac{2}{x-4}}$$

The form of the Partial fraction decomposition.

(2)
of 2

ex:

$$\frac{x-12}{x(x-4)} = \frac{A}{x} + \frac{B}{x-4}$$

$$\frac{x^2+5x+2}{(x-1)(x+2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3}$$

$$\frac{x^2+1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$$\frac{2x^3+7x+5}{(x^2+x+2)(x^2+1)} = \frac{Ax+B}{x^2+x+2} + \frac{Cx+D}{x^2+1}$$

$$\frac{x^4+x^3+x^2-x+1}{x(x^2+1)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$$