1.3 Angular and linear velocity (briefly)

Main ideas: radian $\leftrightarrow$ degree $\boxed{\pi \text{ radians} = 180 \text{ degrees}}$

radian $\leftrightarrow$ revolution $\boxed{2\pi \text{ radians} = 1 \text{ revolution}}$

$= 360$ degrees

example: diameter $= 250$ ft so $r = 125$ ft

2 revolutions/hour. Velocity in ft/sec?

In one hour

$$\alpha = 2 \text{ revolutions} \cdot \frac{2\pi \text{ radians}}{1 \text{ revolution}} = 4\pi \text{ radians}$$

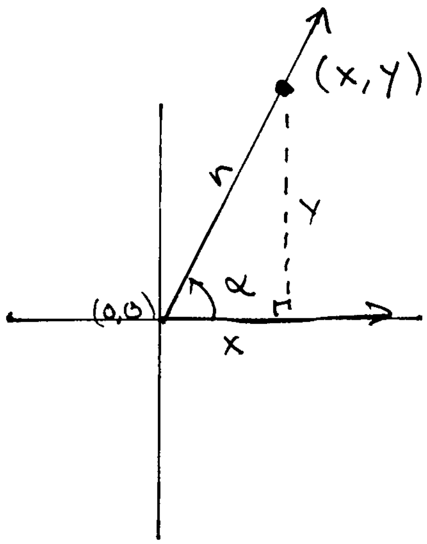
Now use $s = \alpha r = (4\pi)(125 \text{ ft}) = 500\pi$ ft in hour.

$$\frac{500\pi \text{ ft}}{\cancel{\text{hr}}} \cdot \frac{1 \cancel{\text{hr}}}{60 \text{ min}} = \frac{500\pi \text{ ft}}{60 \text{ min}} = \frac{25\pi}{3} \text{ ft/min}$$

$$\frac{25\pi \text{ ft}}{3 \cancel{\text{min}}} \cdot \frac{1 \cancel{\text{min}}}{60 \text{ sec}} = \frac{25\pi \text{ ft}}{180 \text{ sec}} = \frac{5\pi}{36} \text{ ft/sec}$$

≈ 0.436 ft/s

1.4 Ratio definition of the trig functions



Choose (x,y) to be any point the terminal ray. Then

$$r = \sqrt{x^2+y^2}$$

Define:

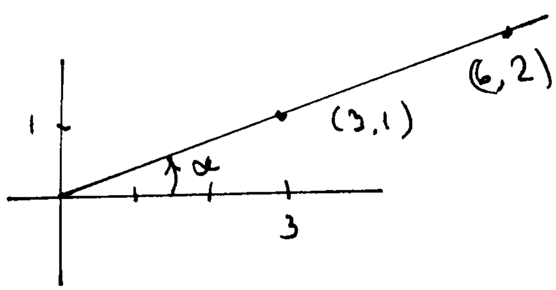
$$\sin\alpha = \frac{y}{r} \qquad \csc\alpha = \frac{r}{y}$$

$$\cos\alpha = \frac{x}{r} \qquad \sec\alpha = \frac{r}{x}$$

$$\tan\alpha = \frac{y}{x} \qquad \cot\alpha = \frac{x}{y}$$

[= slope of the terminal ray]

ex: The terminal ray passes through $(3,1)$. Find all six trig functions.



$$r = \sqrt{3^2+1^2} = \sqrt{10}$$

$$\sin\alpha = \frac{y}{r} = \frac{1}{\sqrt{10}} = \frac{1}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} = \boxed{\frac{\sqrt{10}}{10}} \qquad \csc\alpha = \boxed{\sqrt{10}}$$

$$\cos\alpha = \frac{x}{r} = \frac{3}{\sqrt{10}} = \boxed{\frac{3\sqrt{10}}{10}} \qquad \sec\alpha = \boxed{\frac{\sqrt{10}}{3}}$$

$$\tan\alpha = \frac{y}{x} = \boxed{\frac{1}{3}} \qquad \cot\alpha = \boxed{3}$$

Remark: Had we used $(x,y) = (6,2)$, we would have $x=6$, $y=2$, $r = 2\sqrt{10}$

But the ratios would be the same: $\sin\alpha = \frac{y}{r} = \frac{2}{2\sqrt{10}} = \frac{1}{\sqrt{10}}$

Signs of the trig functions

$\sin\alpha > 0$ $\cos\alpha < 0$ $\tan\alpha < 0$	$\sin\alpha > 0$ $\cos\alpha > 0$ $\tan\alpha > 0$
$\sin\alpha < 0$ $\cos\alpha < 0$ $\tan\alpha > 0$	$\sin\alpha < 0$ $\cos\alpha > 0$ $\tan\alpha < 0$

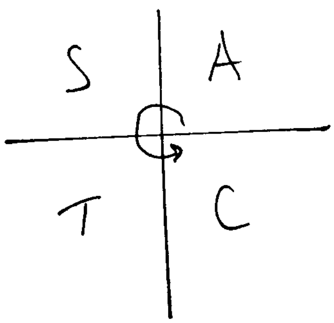
In QI: $x > 0$, $y > 0$

In QII: $x < 0$, $y > 0$

In QIII: $x < 0$, $y < 0$

In QIV: $x > 0$, $y < 0$

In any quadrant, $r > 0$.



All
Students
Take
Calculus.

← Memorize this!

Value of Trig functions of famous angles

Quadrantal angles:

ex: $\sin 0^\circ = \frac{y}{r} = \frac{0}{1} = 0$

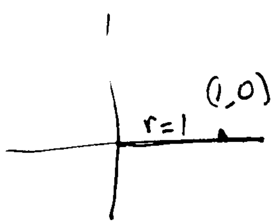
$\cos 0^\circ = x/r = 1/1 = 1$

$\tan 0^\circ = y/x = 0/1 = 0$

$\csc 0^\circ$ is undefined

$\sec 0^\circ = 1$

$\cot 0^\circ$ is undefined



$\sin 90^\circ = \frac{y}{r} = 1$

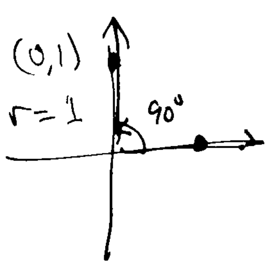
$\cos 90^\circ = \frac{x}{r} = 0$

$\tan 90^\circ = \frac{y}{x}$ is undefined

$\csc 90^\circ = 1$

$\sec 90^\circ$ is undefined $\left[= \frac{r}{x}\right]$

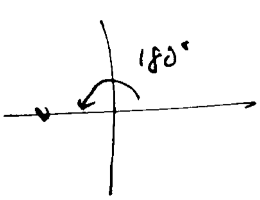
$\cot 90^\circ = \frac{x}{y} = \frac{0}{1} = 0$



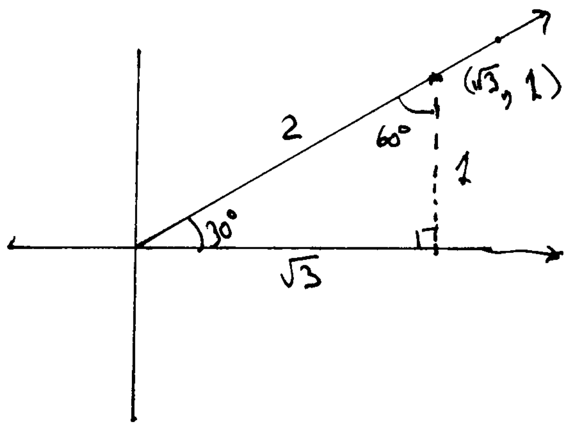
$\sin 180^\circ = 0$

$\cos 180^\circ = -1$

$\tan 180^\circ = 0$



Multiples of 30°

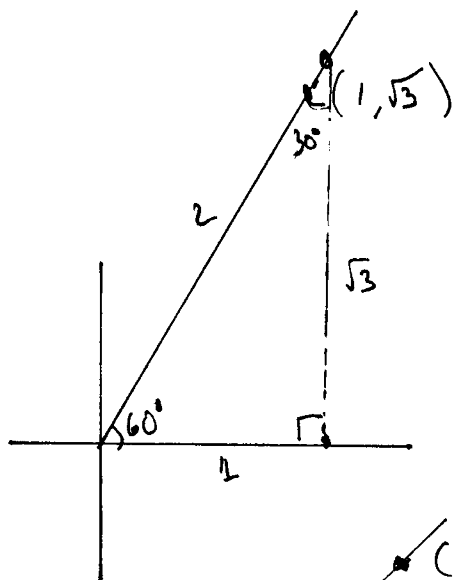


Since
$r = 2$
$x = \sqrt{3}$
$y = 1$
it follows that

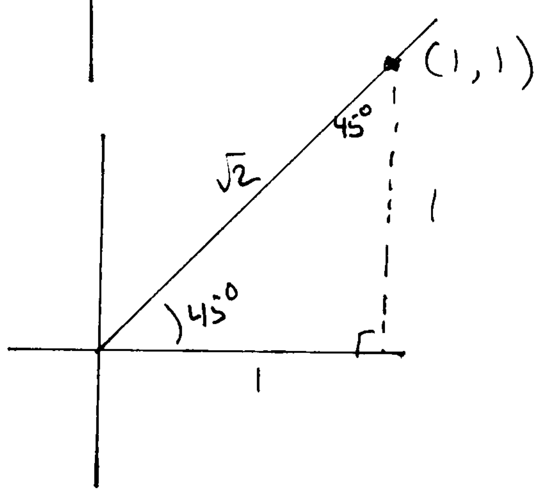
$\sin 30° = \frac{y}{r} = \frac{1}{2}$ $\qquad \csc 30° = 2$

$\cos 30° = \frac{x}{r} = \frac{\sqrt{3}}{2}$ $\qquad \sec 30° = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\tan 30° = \frac{y}{x} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\qquad \cot 30° = \sqrt{3}$



$\sin 60° = \frac{\sqrt{3}}{2}$
$\cos 60° = \frac{1}{2}$
$\tan 60° = \sqrt{3}$



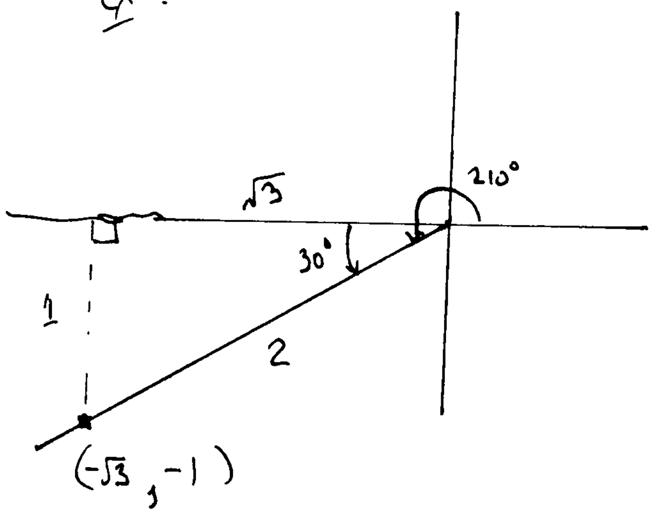
$\sin 45° = \frac{y}{r} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos 45° = \frac{x}{r} = \frac{\sqrt{2}}{2}$

$\tan 45° = 1$

M104 2/9/15

ex: $210° = 30° + 180°$



In QIII

$\sin 210° < 0$

$\cos 210° < 0$

$\tan 210° > 0$

$\sin 210° = \frac{y}{r} = \frac{-1}{2} \quad = -\sin 30°$

$\cos 210° = \frac{x}{r} = -\sqrt{3}/2 \quad = -\cos 30°$

$\tan 210° = \frac{y}{x} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \quad = +\tan 30°$

↑ Aha! These are

$\pm$ sin (reference angle)

or $\pm$ cos (ref. angle)

or $\pm$ tan (ref. angle)